出 版 前 言

2011 年 12 月 11 日是西安交通大学杰出校友钱学森先生的百年诞辰。为缅怀钱学森学长，学习他的科学思想和卓越风范，展示其丰功伟绩和人格魅力，西安交通大学举办了“纪念钱学森诞辰 100 周年”系列活动：作为制片方之一，参与西部电影集团摄制传记故事片《钱学森》；与中央电视台合作，出品纪录片《实验班的故事——沿着钱学森走过的路》；扩建钱学森生平业绩展馆，向校内外开放；举办钱学森科学与教育思想研讨会；出版发行《钱学森力学手稿》、《钱学森年谱（初编）》、《钱学森第六次产业革命思想探微丛书》等。

钱学森先生在美国深造和工作期间留下大量珍贵手稿，这些手稿真实展示了钱学森先生博大精深的学识、开拓求实的精神和严谨奋进的作风，是钱老勇攀科学高峰和严谨治学的集中体现。这里，我们将部分原稿整理汇集成册，出版《钱学森力学手稿》，作为钱老百年诞辰的献礼。

《钱学森力学手稿》共 10 卷，包含两部分内容。第一部分是草稿，包括扁壳、球壳和圆柱壳屈曲分析的公式推导和数值演算。在研究圆柱壳轴压屈曲问题时，为了求得圆柱壳体的临界压力，在有关的五百多页草稿中，对多达二十多种可能的屈曲模

态逐一进行公式推演和数值计算，最终才找到满意的并在论文中采用的屈曲模态。仔细观察草稿中的数据列表，每个数字有效位数都长达八位，在手摇机械式计算机作为主要计算工具的年代，这串串数字凝聚着多少现今难以想象的艰辛劳动。

第二部分是手稿，以航空航天工程为核心，涵盖空气动力学、固体力学、火箭技术、工程控制论和物理力学等领域的部分学术论文手稿、打印稿和讲义。

《钱学森力学手稿》是在西安交通大学校领导的大力支持下，由西安交通大学航天航空学院沈亚鹏教授整理完成。图书出版过程中得到了西安交通大学党委宣传部、校友关系发展部、图书馆、航天航空学院等的积极协助，在此深表感谢。

Preliminary Calculation of Circular Cylinder (Ⅳ)

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We have the equation

$$\frac{\partial^2 v}{\partial x^2}+2\frac{\partial^2 v}{\partial y^2}=-\left\{\frac{\partial w}{\partial y}\left(\frac{\partial^2 w}{\partial x^2}+2\frac{\partial^2 w}{\partial y^2}\right)+\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x\partial y}\right\}$$

$$\frac{w}{R}=\frac{1}{2}\left(\frac{a}{R}\right)^2\left[1-\left(\frac{x}{a}\right)^2-\frac{f}{2}\cos\frac{\pi x}{2a}\left(1+\cos\frac{\pi y}{b}\right)\right]$$

$$\frac{w_0}{R}=\frac{1}{2}\left(\frac{a}{R}\right)^2\left[1-\left(\frac{x}{a}\right)^2\right]$$

$$\frac{\partial w}{\partial x}=\left(\frac{a}{R}\right)\left[-\left(\frac{x}{a}\right)+\frac{f}{8}\pi\sin\frac{\pi x}{2a}\left(1+\cos\frac{\pi y}{b}\right)\right]$$

$$\frac{\partial w}{\partial y}=\left(\frac{a}{R}\right)\left[\qquad+\frac{f}{4}\pi\lambda\cos\frac{\pi x}{2a}\sin\frac{\pi y}{b}\right]$$

$$\frac{\partial^2 w}{\partial x^2}=\frac{1}{R}\left[-1+\frac{f}{16}\pi^2\cos\frac{\pi x}{2a}\left(1+\cos\frac{\pi y}{b}\right)\right]$$

$$\frac{\partial^2 w}{\partial x\partial y}=\frac{1}{R}\left[\qquad-\frac{f}{8}\pi^2\lambda\sin\frac{\pi x}{2a}\sin\frac{\pi y}{b}\right]$$

$$\frac{\partial^2 w}{\partial y^2}=\frac{1}{R}\left[\qquad+\frac{f}{4}\pi^2\lambda^2\cos\frac{\pi x}{2a}\cos\frac{\pi y}{b}\right]$$

$$R\left[\frac{\partial^2 v}{\partial x^2}+2\frac{\partial^2 v}{\partial y^2}\right]=-\left(\frac{a}{R}\right)\left(\frac{f}{4}\pi\lambda\right)\Bigg[\cos\frac{\pi x}{2a}\sin\frac{\pi y}{b}\Big\{-1+\frac{f\pi^2}{16}\cos\frac{\pi x}{2a}\left(1+\cos\frac{\pi y}{b}\right)+\frac{f\pi^2}{2}\lambda^2\cos\frac{\pi x}{2a}\cos\frac{\pi y}{b}\Big\}$$

$$-\frac{\pi}{2}\sin\frac{\pi x}{2a}\sin\frac{\pi y}{b}\Big\{-\left(\frac{x}{a}\right)+\frac{f\pi}{8}\sin\frac{\pi x}{2a}\left(1+\cos\frac{\pi y}{b}\right)\Big\}\Bigg]$$

$$=-\left(\frac{a}{R}\right)\frac{f}{4}\pi\lambda\Bigg[-\cos\frac{\pi x}{2a}\sin\frac{\pi y}{b}+\frac{f\pi^2}{64}\left(1+\cos\frac{\pi x}{a}\right)\left(2\sin\frac{\pi y}{b}+\sin\frac{2\pi y}{b}\right)$$

$$+\left(\frac{\pi x}{2a}\right)\sin\frac{\pi x}{2a}\sin\frac{\pi y}{b}+\frac{f\pi^2}{64}\left(-1+\cos\frac{\pi x}{a}\right)\left(2\sin\frac{\pi y}{b}+\sin\frac{2\pi y}{b}\right)$$

$$+\frac{f\pi^2}{8}\lambda^2\left(1+\cos\frac{\pi x}{a}\right)\sin\frac{2\pi y}{b}\Bigg]$$

$$R\left[\frac{\partial^2 v}{\partial x^2}+2\frac{\partial^2 v}{\partial y^2}\right]=-\left(\frac{a}{R}\right)\frac{f\pi\lambda}{4}\left[\left(\frac{\pi x}{2a}\right)\sin\frac{\pi x}{2a}\sin\frac{\pi y}{b}-\cos\frac{\pi x}{2a}\sin\frac{\pi y}{b}+\frac{f\pi^2}{16}\cos\frac{\pi x}{a}\sin\frac{\pi y}{b}\right.$$

$$\left.+\frac{f\pi^2}{32}\cos\frac{\pi x}{a}\sin\frac{2\pi y}{b}+\frac{f\pi^2}{8}\lambda^2\sin\frac{2\pi y}{b}+\frac{f\pi^2}{8}\lambda^2\cos\frac{\pi x}{a}\sin\frac{2\pi y}{b}\right]$$

$$=-\left(\frac{a}{R}\right)\frac{f\pi\lambda}{4}\left[\left(\frac{\pi x}{2a}\right)\sin\frac{\pi x}{2a}\sin\frac{\pi y}{b}-\cos\frac{\pi x}{2a}\sin\frac{\pi y}{b}+\frac{f\pi^2}{8}\lambda^2\sin\frac{2\pi y}{b}+\frac{f\pi^2}{16}(1+2\lambda^2)\cos\frac{\pi x}{a}\sin\frac{\pi y}{b}\right.$$

$$\left.+\frac{f\pi^2}{32}\cos\frac{\pi x}{a}\sin\frac{2\pi y}{b}\right]$$

Investigate the particular solution of the following eqn.

$$\frac{\partial^2 v}{\partial x^2}+2\frac{\partial^2 v}{\partial y^2}=\left(\frac{\pi x}{2a}\right)\sin\frac{\pi x}{2a}\sin\frac{\pi y}{b}$$

$$B=-\frac{1}{\left(\frac{\pi}{a}\right)^2}\frac{4}{(1+8\lambda^2)}$$

Put $\quad v=X\sin\frac{\pi y}{b}$

$$A=-\frac{1}{\left(\frac{\pi}{a}\right)^2}\frac{8}{(1+8\lambda^2)^2}$$

$$\frac{d^2X}{dx^2}-2\left(\frac{\pi}{b}\right)^2X=\left(\frac{\pi x}{2a}\right)\sin\frac{\pi x}{2a}$$

Let $\quad X=A\cos\frac{\pi x}{2a}+B\left(\frac{\pi x}{2a}\right)\sin\frac{\pi x}{2a}$

$$\frac{d^2X}{dx^2}=\left(\frac{\pi}{a}\right)^2\left\{\frac{(2B-A)}{4}\cos\frac{\pi x}{2a}-\frac{B}{4}\left(\frac{\pi x}{2a}\right)\sin\frac{\pi x}{2a}\right\}$$

$$-2\left(\frac{\pi}{b}\right)^2X=\left(\frac{\pi}{a}\right)^2\left\{-2\lambda^2A\cos\frac{\pi x}{2a}-2\lambda^2B\left(\frac{\pi x}{2a}\right)\sin\frac{\pi x}{2a}\right\}$$

$$\therefore\ \left(\frac{\pi}{a}\right)^2\left\{\frac{2B-A}{4}-2\lambda^2A\right\}=0,\qquad\left(\frac{\pi}{a}\right)^2\left\{-\frac{B}{4}-2\lambda^2B\right\}=1$$

$$\frac{v}{R} = \left(\frac{a}{R}\right)^3 \frac{f}{4}\frac{1}{\pi}\lambda\Big[\frac{8}{(1+8\lambda^2)^2}\cos\frac{\pi x}{2a}\sin\frac{\pi y}{b} + \frac{4}{1+8\lambda^2}\left(\frac{\pi x}{2a}\right)\sin\frac{\pi x}{2a}\sin\frac{\pi y}{b} - \frac{4}{(1+8\lambda^2)}\cos\frac{\pi x}{2a}\sin\frac{\pi y}{b}$$

$$+ \frac{f\pi^2}{8}\lambda^2\frac{1}{8\lambda^2}\sin\frac{2\pi y}{b} + \frac{f\pi^2}{16}\frac{1+2\lambda^2}{1+2\lambda^2}\cos\frac{\pi x}{a}\sin\frac{\pi y}{b} + \frac{f\pi^2}{32}\frac{1}{1+8\lambda^2}\cos\frac{\pi x}{a}\sin\frac{2\pi y}{b}$$

$$+ a_0\left(\frac{\pi y}{b}\right) + a_2\cosh\sqrt{2}\frac{\pi\lambda x}{a}\sin\frac{\pi y}{b}\Big]$$

$$= \left(\frac{a}{R}\right)^3 \frac{f}{4}\frac{1}{\pi}\lambda\Big[\frac{4(1-8\lambda^2)}{(1+8\lambda^2)^2}\cos\frac{\pi x}{2a}\sin\frac{\pi y}{b} + \frac{4}{1+8\lambda^2}\left(\frac{\pi x}{2a}\right)\sin\frac{\pi x}{2a}\sin\frac{\pi y}{b} + \frac{f\pi^2}{64}\sin\frac{2\pi y}{b}$$

$$+ \frac{f\pi^2}{16}\cos\frac{\pi x}{a}\sin\frac{\pi y}{b} + \frac{f\pi^2}{32}\frac{1}{(1+8\lambda^2)}\cos\frac{\pi x}{a}\sin\frac{2\pi y}{b} + a_0\left(\frac{\pi y}{b}\right) + a_2\cosh\frac{\sqrt{2}\pi\lambda x}{a}\sin\frac{\pi y}{b}\Big]$$

$$\frac{\partial v}{\partial x} = \left(\frac{a}{R}\right)^2 \frac{f}{4}\lambda\Big[-\frac{2(1-8\lambda^2)}{(1+8\lambda^2)^2}\sin\frac{\pi x}{2a}\sin\frac{\pi y}{b} + \frac{2}{1+8\lambda^2}\cdot\sin\frac{\pi x}{2a}\sin\frac{\pi y}{b} + \frac{2}{(1+8\lambda^2)}\left(\frac{\pi x}{2a}\right)\cos\frac{\pi x}{2a}\sin\frac{\pi y}{b}$$

$$- \frac{f\pi^2}{16}\sin\frac{\pi x}{a}\sin\frac{\pi y}{b} - \frac{f\pi^2}{32}\frac{1}{(1+8\lambda^2)}\sin\frac{\pi x}{a}\sin\frac{2\pi y}{b} + a_2\sqrt{2}\lambda\sinh\frac{\sqrt{2}\pi\lambda x}{a}\sin\frac{\pi y}{b}\Big]$$

$$\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} = \left(\frac{a}{R}\right)^2 \frac{f}{4}\lambda\Big[-2\left(\frac{\pi x}{2a}\right)\cos\frac{\pi x}{2a}\sin\frac{\pi y}{b} + \frac{f\pi^2}{32}\sin\frac{\pi x}{a}\left(2\sin\frac{\pi y}{b} + \sin\frac{2\pi y}{b}\right)\Big]$$

$$\left(\frac{a}{R}\right)^2 \frac{f}{4} \lambda^2 a_0 = -\frac{\sigma}{E} - \left(\frac{a}{R}\right)^2 \frac{f}{4} \lambda^2 \left[-\frac{16}{\pi} \frac{1}{(1+8\lambda^2)^2} + \frac{f\pi^2}{32} + \frac{16}{\pi} \frac{1}{(1+8\lambda^2)^2} \right]$$

$$= -\frac{\sigma}{E} - \left(\frac{a}{R}\right)^2 \frac{f}{4} \lambda^2 \left(\frac{f\pi^2}{32}\right)$$

$$\boxed{\frac{\Delta p}{abtE} = -\left(\frac{\sigma}{E}\right)^2 - \left(\frac{\sigma}{E}\right)\left(\frac{a}{R}\right)^2 \left(\frac{f}{4}\right)^2 \left(\frac{\pi^2 \lambda^2}{8}\right)}$$

$$\varepsilon_1 = \frac{1}{2}\left\{\left(\frac{\partial w}{\partial x}\right)^2 - \left(\frac{\partial w_0}{\partial x}\right)^2\right\} = \left(\frac{a}{R}\right)^2 \frac{f}{4} \left\{ -\left(\frac{\pi x}{2a}\right) \sin\frac{\pi x}{2a}\left(1+\cos\frac{\pi y}{b}\right) + \frac{f\pi^2}{128}\left(1-\cos\frac{\pi x}{a}\right)\left(3+4\cos\frac{\pi y}{b} + \cos\frac{2\pi y}{b}\right) \right\}$$

$$= \left(\frac{a}{R}\right)^2 \frac{f}{4} \left\{ -\left(\frac{\pi x}{2a}\right) \sin\frac{\pi x}{2a} + \frac{3f\pi^2}{128}\left(1-\cos\frac{\pi x}{a}\right) \right.$$

$$+ \left\{ -\left(\frac{\pi x}{2a}\right)\sin\frac{\pi x}{2a} + \frac{f\pi^2}{32}\left(1-\cos\frac{\pi x}{a}\right)\cos\frac{\pi y}{b} \right.$$

$$\left. + \frac{f\pi^2}{128}\left(1-\cos\frac{\pi x}{a}\right)\cos\frac{2\pi y}{b} \right\}$$

$$\frac{1}{2ab}\int_0^a\int_0^b \varepsilon_1^2\, dx\,dy = \left(\frac{a}{R}\right)^4 \left(\frac{f}{4}\right)^2 \left\{ \frac{3}{4}\left[\left(\frac{3}{128}\right)^2 + \frac{1}{2}\left(\frac{1}{32}\right)^2 + \frac{1}{2}\left(\frac{1}{128}\right)^2\right](f\pi^2)^2 + \frac{3}{4}\int_0^1 \left(\frac{\pi x}{2a}\right)^2 \sin^2\frac{\pi x}{2a}\, d\left(\frac{x}{a}\right) \right.$$

$$\left. - \frac{5}{128}(f\pi^2)\int_0^1 \left(\frac{\pi x}{2a}\right)\sin\frac{\pi x}{2a}\left(1-\cos\frac{\pi x}{a}\right) d\left(\frac{x}{a}\right) \right\}$$

$$= \left(\frac{a}{R}\right)^4 \left(\frac{f}{4}\right)^2 \left[\frac{105}{8(128)^2}(f\pi^2)^2 - \frac{5}{128}\left(\frac{2}{\pi} + \frac{10}{9\pi}\right)(f\pi^2) + \left(\frac{\pi^2}{32} + \frac{3}{16}\right) \right] \quad !$$

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$$\frac{\partial v}{\partial y} = \left(\frac{a}{R}\right)^2 \frac{f}{4}\lambda^2 \Big[\frac{4(1-\delta\lambda^2)}{(1+\delta\lambda^2)^2} \cos\frac{\pi x}{2a} \cos\frac{\pi y}{b} + \frac{4}{(1+\delta\lambda^2)} \left(\frac{\pi x}{2a}\right) \sin\frac{\pi x}{2a} \cos\frac{\pi y}{b} + \frac{f\pi^2}{32} \cos\frac{2\pi y}{b}$$

$$+ \frac{f\pi^2}{16} \cos\frac{\pi x}{a} \cos\frac{\pi y}{b} + \frac{f\pi^2}{16} \frac{1}{(1+\delta\lambda^2)} \cos\frac{\pi x}{a} \cos\frac{2\pi y}{b} - \frac{f\pi^2}{32} + a_2 \cosh\frac{\sqrt{2}\,\pi\lambda x}{a} \cos\frac{\pi y}{b} \Big] - \frac{\sigma}{E}$$

$$\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 = \left(\frac{a}{R}\right)^2 \frac{f}{4}\lambda^2 \left[\frac{f\pi^2}{32} \left(1+\cos\frac{\pi x}{a}\right)\left(1-\cos\frac{2\pi y}{b}\right) \right] = \left(\frac{a}{R}\right)^2 \frac{f}{4}\lambda^2 \Big[\frac{f\pi^2}{32} + \frac{f\pi^2}{32}\cos\frac{\pi x}{a} - \frac{f\pi^2}{32}\cos\frac{2\pi y}{b}$$

$$- \frac{f\pi^2}{32} \cos\frac{\pi x}{a} \cos\frac{2\pi y}{b} \Big]$$

$$\varepsilon_2 = \left(\frac{a}{R}\right)^2 \frac{f}{4}\lambda^2 \Big[\frac{4(1-\delta\lambda^2)}{(1+\delta\lambda^2)^2} \cos\frac{\pi x}{2a} \cos\frac{\pi y}{b} + \frac{4}{(1+\delta\lambda^2)} \left(\frac{\pi x}{2a}\right) \sin\frac{\pi x}{2a} \cos\frac{\pi y}{b} + \frac{f\pi^2}{16} \cos\frac{\pi x}{a} \cos\frac{\pi y}{b} + \frac{f\pi^2}{32} \cos\frac{\pi x}{a}$$

$$+ \frac{f\pi^2}{32} \left(\frac{1-\delta\lambda^2}{1+\delta\lambda^2}\right) \cos\frac{\pi x}{a} \cos\frac{2\pi y}{b} + a_2 \cosh\frac{\sqrt{2}\,\pi\lambda x}{a} \cos\frac{\pi y}{b} \Big] - \frac{\sigma}{E}$$

$$= \left(\frac{a}{R}\right)^2 \frac{f}{4}\lambda^2 \Big[\frac{f\pi^2}{32} \cos\frac{\pi x}{a}$$

$$+ \left\{ \frac{4(1-\delta\lambda^2)}{(1+\delta\lambda^2)^2} \cos\frac{\pi x}{2a} + \frac{f\pi^2}{16} \cos\frac{\pi x}{a} + \frac{4}{1+\delta\lambda^2} \left(\frac{\pi x}{2a}\right) \sin\frac{\pi x}{2a} + a_2 \cosh\frac{\sqrt{2}\,\pi\lambda x}{a} \right\} \cos\frac{\pi y}{b}$$

$$+ \left\{ \frac{f\pi^2}{32} \left(\frac{1-\delta\lambda^2}{1+\delta\lambda^2}\right) \cos\frac{\pi x}{a} \right\} \cos\frac{2\pi y}{b} \Big] - \frac{\sigma}{E}$$

$$\frac{1}{2ab}\int_0^a\int_0^b \varepsilon_2^2\,dx\,dy = \left(\frac{a}{R}\right)^4\left(\frac{f}{4}\right)^2\lambda^4\left[\frac{1}{4}\left(\frac{f\pi^2}{32}\right)^2 + \frac{1}{8}\left\{\frac{4(1-\delta\lambda^2)}{(1+\delta\lambda^2)^2}\right\}^2 + \frac{1}{8}\left(\frac{f\pi^2}{16}\right)^2\right.$$

$$+\ \frac{2(1-\delta\lambda^2)}{(1+\delta\lambda^2)^2}\int_0^1 \cos\frac{\pi x}{2a}\left\{\frac{4}{1+\delta\lambda^2}\left(\frac{\pi x}{2a}\right)\sin\frac{\pi x}{2a} + a_2\cosh\frac{\sqrt{2}\pi\lambda x}{a}\right\}d\left(\frac{x}{a}\right)$$

$$+\ \frac{f\pi^2}{32}\int_0^1 \cos\frac{\pi x}{a}\left\{\frac{4}{1+\delta\lambda^2}\left(\frac{\pi x}{2a}\right)\sin\frac{\pi x}{2a} + a_2\cosh\frac{\sqrt{2}\pi\lambda x}{a}\right\}d\left(\frac{x}{a}\right)$$

$$+\ \frac{1}{4}\int_0^1\left\{\frac{4}{1+\delta\lambda^2}\left(\frac{\pi x}{2a}\right)\sin\frac{\pi x}{2a} + a_2\cosh\frac{\sqrt{2}\pi\lambda x}{a}\right\}^2 d\left(\frac{x}{a}\right)$$

$$\left.+\ \frac{1}{8}\left\{\frac{f\pi^2}{32}\,\frac{1-\delta\lambda^2}{1+\delta\lambda^2}\right\}^2\right] + \frac{1}{2}\left(\frac{\sigma}{E}\right)^2$$

$$\frac{1}{4ab}\int_0^a\int_0^b \gamma^2\,dx\,dy = \left(\frac{a}{R}\right)^2\left(\frac{f}{4}\right)^2\lambda^2\left[\frac{1}{4}\left\{\frac{16\lambda^2}{(1+\delta\lambda^2)^2}\right\}^2 + \frac{16\lambda^2}{(1+\delta\lambda^2)^2}\int_0^1 \sin\frac{\pi x}{2a}\left\{-\frac{\delta\lambda^2}{1+\delta\lambda^2}\left(\frac{\pi x}{2a}\right)\cos\frac{\pi x}{2a} + \frac{a_2\lambda}{\sqrt{2}}\sinh\frac{\sqrt{2}\pi\lambda x}{a}\right\}d\left(\frac{x}{a}\right)\right.$$

$$+\ \frac{1}{2}\int_0^1\left\{-\frac{\delta\lambda^2}{(1+\delta\lambda^2)}\left(\frac{\pi x}{2a}\right)\cos\frac{\pi x}{2a} + \frac{a_2\lambda}{\sqrt{2}}\sinh\frac{\sqrt{2}\pi\lambda x}{a}\right\}^2 d\left(\frac{x}{a}\right)$$

$$\left.+\ \frac{1}{4}\left(\frac{f\pi^2}{8}\,\frac{\lambda^2}{1+\delta\lambda^2}\right)^2\right]$$

$$\int_0^1 \underline{\left(\frac{\pi x}{2a}\right) \sin\frac{\pi x}{2a}\cos\frac{\pi x}{2a}\, d\left(\frac{x}{a}\right)} = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \theta \sin 2\theta\, d\theta = \frac{1}{4\pi}\int_0^{\pi} x \sin x\, dx = \frac{1}{4\pi}\Big[\sin x - x\cos x\Big]_0^{\pi} = \underline{\frac{1}{4}}$$

$$\int_0^1 \underline{\cos\frac{\pi x}{2a}\cosh\frac{\sqrt{2}\pi x}{a}\, d\left(\frac{x}{a}\right)} = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\Big[\cosh(2\sqrt{2}\lambda + i)\theta + \cosh(2\sqrt{2}\lambda - i)\theta\Big]d\theta$$

$$= \frac{1}{\pi}\left[\frac{\sinh(2\sqrt{2}+i)\frac{\pi}{2}}{2\sqrt{2}\lambda + i} + \frac{\sinh(2\sqrt{2}-i)\frac{\pi}{2}}{2\sqrt{2}\lambda - i}\right] = \underline{\frac{2}{\pi}\,\frac{\cosh\sqrt{2}\lambda\pi}{(1+8\lambda^2)}}$$

$$\int_0^1 \underline{\left(\frac{\pi x}{2a}\right)\sin\frac{\pi x}{2a}\cos\frac{\pi x}{a}\, d\left(\frac{x}{a}\right)} = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \theta\Big[\sin 3\theta - \sin\theta\Big]d\theta = \frac{1}{\pi}\Big[\frac{1}{9}\{\sin 3\theta - 3\theta\cos 3\theta\} - \{\sin\theta - \theta\cos\theta\}\Big]_0^{\frac{\pi}{2}}$$

$$= \underline{-\frac{10}{9\pi}}$$

$$\int_0^1 \underline{\cosh\frac{\sqrt{2}\pi\lambda x}{a}\cos\frac{\pi x}{a}\, d\left(\frac{x}{a}\right)} = \frac{1}{2\pi}\int_0^{\pi}\Big[\cosh(\sqrt{2}\lambda + i)\theta + \cosh(\sqrt{2}\lambda - i)\theta\Big]d\theta$$

$$= \frac{1}{2\pi}\left[\frac{\sinh(\sqrt{2}\lambda + i)\pi}{\sqrt{2}\lambda + i} + \frac{\sinh(\sqrt{2}\lambda - i)\pi}{\sqrt{2}\lambda - i}\right] = \underline{-\frac{1}{\pi}\,\frac{\sqrt{2}\lambda\sinh\sqrt{2}\lambda\pi}{1+2\lambda^2}}$$

$$\int_0^1 \underline{\left(\frac{\pi x}{2a}\right)^2 \sin^2\frac{\pi x}{2a}\, d\left(\frac{x}{a}\right)} = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \theta^2\Big[1 - \cos 2\theta\Big]d\theta = \frac{1}{\pi}\Big[\frac{1}{3}\left(\frac{\pi}{2}\right)^3 - \frac{1}{8}\{-2x\cos x + (x^2-2)\sin x\}_0^{\pi}\Big]$$

$$= \frac{1}{\pi}\Big[\frac{1}{24}\pi^3 + \frac{1}{4}\pi\Big] = \underline{\Big[\frac{\pi^2}{24} + \frac{1}{4}\Big]}$$

$$\int_0^1 \left(\frac{\pi x}{2a}\right) \sin\frac{\pi x}{2a} \cosh\frac{\sqrt{2}\pi\lambda x}{a}\, d\left(\frac{x}{a}\right) = \frac{1}{\pi i}\int_0^{\frac{\pi}{2}} \theta \left[\sinh(2\sqrt{2}\lambda + i)\theta - \sinh(2\sqrt{2}\lambda - i)\theta \right] d\theta$$

$$= \frac{1}{\pi i}\left[\frac{\frac{\pi}{2}}{2\sqrt{2}\lambda + i} \cosh(2\sqrt{2}\lambda + i)\frac{\pi}{2} - \frac{\frac{\pi}{2}}{2\sqrt{2}\lambda - i} \cosh(2\sqrt{2}\lambda - i)\frac{\pi}{2} \right.$$

$$\left. - \left\{ \frac{1}{(2\sqrt{2}\lambda + i)^2} \sinh(2\sqrt{2}\lambda + i)\frac{\pi}{2} - \frac{1}{(2\sqrt{2}\lambda - i)^2} \sinh(2\sqrt{2}\lambda - i)\frac{\pi}{2} \right\} \right]$$

$$= \frac{1}{\pi}\left[\frac{2\sqrt{2}\pi\lambda \sinh\sqrt{2}\lambda\pi}{1 + 8\lambda^2} + \frac{2(1 - 8\lambda^2)\cosh\sqrt{2}\lambda\pi}{(1 + 8\lambda^2)^2} \right]$$

$$\int_0^1 \cosh^2\frac{\sqrt{2}\pi\lambda x}{a}\, d\left(\frac{x}{a}\right) = \frac{1}{2}\int_0^1 \left[1 + \cosh\frac{2\sqrt{2}\pi\lambda x}{a} \right] d\left(\frac{x}{a}\right) = \frac{1}{2} + \frac{\sinh 2\sqrt{2}\lambda\pi}{4\sqrt{2}\pi\lambda}$$

$$\int_0^1 \left(\frac{\pi x}{2a}\right) \sin\frac{\pi x}{2a} \cos\frac{\pi x}{2a}\, d\left(\frac{x}{a}\right) = \frac{1}{4}$$

$$\int_0^1 \sin\frac{\pi x}{2a} \sinh\frac{\sqrt{2}\pi\lambda x}{a}\, d\left(\frac{x}{a}\right) = \frac{1}{\pi i}\int_0^{\frac{\pi}{2}} \left[\cosh(2\sqrt{2}\lambda + i)\theta - \cosh(2\sqrt{2}\lambda - i)\theta \right] d\theta$$

$$= \frac{1}{\pi i}\left[\frac{\sinh(2\sqrt{2}\lambda + i)\frac{\pi}{2}}{2\sqrt{2}\lambda + i} - \frac{\sinh(2\sqrt{2}\lambda - i)\frac{\pi}{2}}{2\sqrt{2}\lambda - i} \right] = \frac{1}{\pi}\,\frac{4\sqrt{2}\lambda \cosh\sqrt{2}\lambda\pi}{(1 + 8\lambda^2)}$$

$$\int_0^1 \left(\frac{\pi x}{2a}\right)^2 \cos^2\frac{\pi x}{2a}\, d\left(\frac{x}{a}\right) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \theta^2 \left[1 + \cos 2\theta \right] d\theta = \left[\frac{\pi^2}{24} - \frac{1}{4} \right]$$

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$$\int_0^1 \frac{\pi x}{2a} \cos\frac{\pi x}{2a} \sinh\frac{\sqrt{2}\pi\lambda x}{a}\, d\left(\frac{x}{a}\right) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \theta\left[\sinh(2\sqrt{2}\lambda+i)\theta + \sinh(2\sqrt{2}\lambda-i)\theta\right] d\theta$$

$$= \frac{1}{\pi}\left[\frac{\frac{\pi}{2}}{2\sqrt{2}\lambda+i}\cosh(2\sqrt{2}\lambda+i)\frac{\pi}{2} + \frac{\frac{\pi}{2}}{2\sqrt{2}\lambda-i}\cosh(2\sqrt{2}\lambda-i)\frac{\pi}{2}\right]$$

$$-\frac{1}{\pi}\left[\frac{1}{(2\sqrt{2}\lambda+i)^2}\sinh(2\sqrt{2}\lambda+i)\frac{\pi}{2} + \frac{1}{(2\sqrt{2}\lambda-i)^2}\sinh(2\sqrt{2}\lambda-i)\frac{\pi}{2}\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi \sinh\sqrt{2}\pi\lambda}{1+8\lambda^2} - \frac{8\sqrt{2}\lambda\cosh\sqrt{2}\lambda\pi}{(1+8\lambda^2)^2}\right]$$

$$\int_0^1 \sinh^2\frac{\sqrt{2}\pi\lambda x}{a}\, d\left(\frac{x}{a}\right) = \frac{\sinh 2\sqrt{2}\lambda\pi}{4\sqrt{2}\pi\lambda} - \frac{1}{2}$$

$$\frac{1}{2ab}\int_0^a\int_0^b \varepsilon_2^2\,dx\,dy = \left(\frac{a}{R}\right)^4\left(\frac{f}{4}\right)^2\lambda^4\left[\frac{1}{4}\left(\frac{f\pi^2}{32}\right)^2 + 2\frac{(1-8\lambda^2)^2}{(1+8\lambda^2)^4} + \frac{1}{8}\left(\frac{f\pi^2}{16}\right)^2\right.$$

$$+\frac{2(1-8\lambda^2)}{(1+8\lambda^2)^2}\left\{\frac{1}{1+8\lambda^2} - \frac{64\lambda^2\cosh\sqrt{2}\lambda\pi}{(1+8\lambda^2)^3(\sqrt{2}\lambda\pi\sinh\sqrt{2}\pi\lambda)}\right\} + \frac{f\pi^2}{32}\left\{-\frac{40}{9\pi}\frac{1}{(1+8\lambda^2)} + \frac{32\lambda^2}{(1+2\lambda^2)(1+8\lambda^2)^2\pi}\right\}$$

$$+\frac{1}{4}\left\{\left(\frac{\pi^2}{24}+\frac{1}{4}\right)\frac{16}{(1+8\lambda^2)^2}\right\} - \frac{2}{1+8\lambda^2}\cdot\frac{32\lambda^2}{(1+8\lambda^2)^2}\left\{\frac{2}{1+8\lambda^2} + \frac{2(1-8\lambda^2)}{(1+8\lambda^2)^2}\frac{\cosh\sqrt{2}\lambda\pi}{(\sqrt{2}\lambda\pi\sinh\sqrt{2}\pi\lambda)}\right\}$$

$$\left.+\frac{1}{4}\frac{(32\lambda^2)^2}{(1+8\lambda^2)^4(\sqrt{2}\lambda\sinh\sqrt{2}\pi\lambda)^2}\left\{\frac{1}{2} + \frac{\sinh 2\sqrt{2}\lambda\pi}{4\sqrt{2}\lambda\pi}\right\} + \frac{1}{8}\left\{\frac{f\pi^2}{32}\frac{1-8\lambda^2}{1+8\lambda^2}\right\}^2\right] + \frac{1}{2}\left(\frac{\sigma}{E}\right)^2$$

$$\frac{1}{4ab}\int_0^a\int_0^b \gamma^2\,dx\,dy = \left(\frac{a}{R}\right)^2\left(\frac{f}{4}\right)^2\lambda^2\left[\frac{64\lambda^4}{(1+8\lambda^2)^4} + \frac{16\lambda^2}{(1+8\lambda^2)^2}\left\{-\frac{2\lambda^2}{1+8\lambda^2} - \frac{128\lambda^4\cosh\sqrt{2}\lambda\pi}{(1+8\lambda^2)^3(\sqrt{2}\lambda\pi\sinh\sqrt{2}\pi\lambda)}\right\}\right.$$

$$+\frac{1}{2}\frac{64\lambda^4}{(1+8\lambda^2)^2}\left(\frac{\pi^2}{24}-\frac{1}{4}\right) + \frac{128\lambda^4}{(1+8\lambda^2)^3}\left\{\frac{1}{(1+8\lambda^2)} - \frac{16\lambda^2\cosh\sqrt{2}\lambda\pi}{(1+8\lambda^2)^2(\sqrt{2}\lambda\pi\sinh\sqrt{2}\lambda\pi)}\right\}$$

$$\left.+\frac{\lambda^2}{4}\frac{(32\lambda^2)^2}{(1+8\lambda^2)^4(\sqrt{2}\lambda\sinh\sqrt{2}\lambda\pi)^2}\left\{-\frac{1}{2} + \frac{\sinh 2\sqrt{2}\lambda\pi}{4\sqrt{2}\lambda\pi}\right\} + \frac{1}{4}\left(\frac{f\pi^2}{8}\frac{\lambda^2}{1+8\lambda^2}\right)^2\right]$$

$$\underline{H_1(\lambda)} = \frac{105}{8(128)^2} + \frac{\lambda^4}{4}\left(\frac{1}{32}\right)^2 + \frac{\lambda^4}{8}\left(\frac{1}{16}\right)^2 + \frac{\lambda^4}{8}\left(\frac{1-8\lambda^2}{1+8\lambda^2}\right)^2\left(\frac{1}{32}\right)^2 + \frac{\lambda^2}{4}\left(\frac{1}{8}\frac{\lambda^2}{1+8\lambda^2}\right)^2$$

$$= \frac{105}{8(128)^2} + \frac{3\lambda^4}{(16)^3} + \frac{\lambda^4}{2(16)^3} = \underline{\underline{\frac{105}{(128)^3} + \frac{7\lambda^4}{2(16)^3}}}$$

$$H_2(\lambda) = + \frac{35}{288\pi} + \frac{5\lambda^4}{36\pi(1+8\lambda^2)} - \frac{\lambda^6}{\pi(1+2\lambda^2)(1+8\lambda^2)^2}$$

$$= \underline{\frac{1}{\pi}\left[\frac{35}{288} + \frac{\lambda^4}{(1+8\lambda^2)}\left\{\frac{5}{36} - \frac{\lambda^2}{(1+2\lambda^2)(1+8\lambda^2)}\right\}\right]}$$

$$H_3(\lambda) = \left(\frac{\pi^2}{32} + \frac{3}{16}\right) + \cancel{2\lambda^4\frac{(1-8\lambda^2)^2}{(1+8\lambda^2)^4}} + \frac{2\lambda^4(1-8\lambda^2)}{(1+8\lambda^2)^3} - \cancel{\frac{128\lambda^6(1-8\lambda^2)}{(1+8\lambda^2)^5}g} + \cancel{\frac{\lambda^4}{(1+8\lambda^2)^2}\left(\frac{\pi^2}{6}+1\right)}$$

$$- \cancel{\frac{128\lambda^6}{(1+8\lambda^2)^4}} - \cancel{\frac{128\lambda^6(1-8\lambda^2)}{(1+8\lambda^2)^5}g} + \cancel{\frac{128\lambda^6}{(1+8\lambda^2)^4}g} + \cancel{\frac{64\lambda^6}{(1+8\lambda^2)^4}} - \frac{32\lambda^6}{(1+8\lambda^2)^3}$$

$$- \cancel{\frac{128\times16\lambda^8}{(1+8\lambda^2)^5}g} + \cancel{\frac{8\lambda^6}{(1+8\lambda^2)^2}\left(\frac{\pi^2}{6}-1\right)} + \cancel{\frac{128\lambda^6}{(1+8\lambda^2)^4}} - \cancel{\frac{128\times16\lambda^8}{(1+8\lambda^2)^5}g}$$

$$= \left(\frac{\pi^2}{32} + \frac{3}{16}\right) + \frac{64\lambda^6}{(1+8\lambda^2)^4}g + \frac{\lambda^4}{1+8\lambda^2}\frac{\pi^2}{6} + \frac{\lambda^4(1-8\lambda^2)}{(1+8\lambda^2)^2} + \frac{2\lambda^4}{(1+8\lambda^2)^2} + \frac{2\lambda^4(1-24\lambda^2)}{(1+8\lambda^2)^3}$$

$$g = \frac{\cosh\sqrt{2}\pi\lambda}{(\sqrt{2}\pi\lambda\sinh\sqrt{2}\pi\lambda)}$$

$$\mathcal{E}_2 = \frac{1}{24}\left(\frac{t}{R}\right)^2 \left\{ \left(\frac{f\pi^2}{16}\right)^2 \frac{3}{4} + \left(\frac{f\pi^2}{4}\right)^2 \lambda^4 \frac{1}{4} + 2\left(\frac{f\pi^2}{8}\right)^2 \lambda^2 \frac{1}{4} \right\}$$

$$= \left(\frac{t}{R}\right)^2 \left(\frac{f}{4}\right)^2 \pi^4 \left\{ \frac{1}{512} + \frac{\lambda^4}{96} + \frac{\lambda^2}{192} \right\}$$

Total energy $= \left(\frac{a}{R}\right)^4 \left(\frac{f}{4}\right)^2 \left\{ H_1(\lambda)(f\pi^2)^2 + H_2(\lambda)(f\pi^2) + H_3(\lambda) \right\}$

$$+ \left(\frac{t}{R}\right)^2 \left(\frac{f}{4}\right)^2 \pi^4 \left\{ \frac{1}{512} + \frac{\lambda^2}{192} + \frac{\lambda^4}{96} \right\} - \left(\frac{\sigma}{E}\right)\left(\frac{a}{R}\right)^2 \left(\frac{f}{4}\right)^2 \left(\frac{\pi^2\lambda^2}{8}\right) - \frac{1}{2}\left(\frac{\sigma}{E}\right)$$

Thus

$$\left(\frac{\sigma}{E}\right)\left(\frac{a}{R}\right)^2 \frac{\pi^2\lambda^2}{4} = \left(\frac{a}{R}\right)^4 \left\{ 4H_1(f\pi^2)^2 + 3H_2(f\pi^2) + 2H_3 \right\}$$

$$+ \left(\frac{t}{R}\right)^2 \pi^4 \left\{ \frac{1}{256} + \frac{\lambda^2}{96} + \frac{\lambda^4}{48} \right\}$$

$$\lambda^2 K = \gamma^2 \left\{ 16H_1(f\pi)^2 + 12H_2 f + \frac{8H_3}{\pi^2} \right\} + \frac{1}{\gamma}\pi^2 \left\{ \frac{1}{64} + \frac{\lambda^2}{24} + \frac{\lambda^4}{12} \right\}$$

$$= \frac{\pi^2}{\gamma^2} \left\{ 64H_1\left(\frac{f}{t}\right)^2 + \left[\frac{1}{64} + \frac{\lambda^2}{24} + \frac{\lambda^4}{12} \right] \right\} + 8H_3 \frac{\gamma^2}{\pi^2} + 24H_2\left(\frac{f}{t}\right)$$

$$= 2\left\{ 512 H_1 H_3 \left(\frac{f}{t}\right)^2 + H_3 \left[\frac{1}{8} + \frac{\lambda^2}{3} + \frac{2\lambda^4}{3} \right] \right\}^{\frac{1}{2}} + 24 H_2 \left(\frac{f}{t}\right)$$

$$\gamma_{min}^2 = \pi^2 \left\{ \frac{8H_1}{H_3}\left(\frac{f}{t}\right)^2 + \frac{1}{H_3}\left[\frac{1}{512} + \frac{\lambda^2}{192} + \frac{\lambda^4}{96} \right] \right\}^{\frac{1}{2}}$$

$$\left(\frac{a}{R}\right)^2 = \left(\frac{t}{R}\right)\pi^2 \left\{ \frac{8H_1}{H_3}\left(\frac{f}{t}\right)^2 + \frac{1}{H_3}\left[\frac{1}{512} + \frac{\lambda^2}{192} + \frac{\lambda^4}{96} \right] \right\}^{\frac{1}{2}}$$

$$\frac{\partial^4 F}{\partial x^4} + 2\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = \frac{E}{R^2}\frac{f\pi^2\lambda^2}{4}\Big\{\frac{f\pi^2}{64}\Big[\Big(1-\cos\frac{\pi x}{a}\Big)\Big(1-\cos\frac{2\pi y}{b}\Big) - \Big(1+\cos\frac{\pi x}{a}\Big)\Big(1+2\cos\frac{\pi y}{b}+\cos\frac{2\pi y}{b}\Big)\Big] + \cos\frac{\pi x}{2a}\cos\frac{\pi y}{b}\Big]$$

$$= \frac{E}{R^2}\frac{f\pi^2\lambda^2}{4}\Big\{\frac{f\pi^2}{64}\Big(-2\cos\frac{\pi x}{a} - 2\cos\frac{2\pi y}{b} - 2\cos\frac{\pi y}{b} - 2\cos\frac{\pi x}{a}\cos\frac{\pi y}{b}\Big) + \cos\frac{\pi x}{2a}\cos\frac{\pi y}{b}\Big\}$$

$$= \frac{E}{R^2}\frac{f\pi^2\lambda^2}{4}\Big\{\cos\frac{\pi x}{2a}\cos\frac{\pi y}{b} - \frac{f\pi^2}{32}\Big(\cos\frac{\pi x}{a} + \cos\frac{\pi y}{b} + \cos\frac{2\pi y}{b} + \cos\frac{\pi x}{a}\cos\frac{\pi y}{b}\Big)\Big\}$$

The particular integral

$$F = \frac{E}{R^2}\frac{f\pi^2\lambda^2}{4}\Big\{\frac{\cos\frac{\pi x}{2a}\cos\frac{\pi y}{b}}{\big\{\big(\frac{\pi}{2a}\big)^2+\big(\frac{\pi}{b}\big)^2\big\}^2} - \frac{f\pi^2}{32}\Big[\frac{\cos\frac{\pi x}{a}}{\big(\frac{\pi}{a}\big)^4} + \frac{\cos\frac{\pi y}{b}}{\big(\frac{\pi}{b}\big)^4} + \frac{\cos\frac{2\pi y}{b}}{\big(\frac{2\pi}{b}\big)^4} + \frac{\cos\frac{\pi x}{a}\cos\frac{\pi y}{b}}{\big\{\big(\frac{\pi}{a}\big)^2+\big(\frac{\pi}{b}\big)^2\big\}^2}\Big]\Big\}$$

$$= E\Big(\frac{a}{R}\Big)^2\frac{f\lambda^2}{4}\Big\{\frac{\cos\frac{\pi x}{2a}\cos\frac{\pi y}{b}}{\big(\frac{\pi}{a}\big)^2\big[\frac{1}{4}+\lambda^2\big]^2} - \frac{f\pi^2}{32}\Big[\frac{\cos\frac{\pi x}{a}}{\big(\frac{\pi}{a}\big)^2} + \frac{\cos\frac{\pi y}{b}}{\big(\frac{\pi}{a}\big)^2\lambda^4} + \frac{\cos\frac{2\pi y}{b}}{\big(\frac{\pi}{a}\big)^2 16\lambda^4} + \frac{\cos\frac{\pi x}{a}\cos\frac{\pi y}{b}}{\big(\frac{\pi}{a}\big)^2\big[1+\lambda^2\big]^2}\Big]\Big\}$$

Write the complete solution as

$$F = E\left(\frac{g}{R}\right)^2 \frac{f\lambda^2}{4} \frac{1}{\left(\frac{\pi}{a}\right)^2}\left[\frac{16\cos\frac{\pi x}{2a}\cos\frac{\pi y}{b}}{(1+4\lambda^2)^2} - \frac{f\pi^2}{32}\left\{\cos\frac{\pi x}{a} + \frac{1}{\lambda^4}\cos\frac{\pi y}{b} + \frac{1}{16\lambda^4}\cos\frac{2\pi y}{b}\right.\right.$$

$$\left.+ \frac{\cos\frac{\pi x}{a}\cos\frac{\pi y}{b}}{(1+\lambda^2)^2}\right\} + a_0\left(\frac{\pi x}{a}\right)^2 + \left\{a_1\cosh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\right\}\cos\frac{\pi y}{b}$$

$$\left. + \left\{a_2\cosh\frac{2\pi\lambda x}{a} + b_2\left(\frac{2\pi\lambda x}{a}\right)\sinh\frac{2\pi\lambda x}{a}\right\}\cos\frac{2\pi y}{b}\right]$$

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} = E\left(\frac{g}{R}\right)^2 \frac{f\lambda^2}{4}\left[-\frac{16\lambda^2\cos\frac{\pi x}{2a}\cos\frac{\pi y}{b}}{(1+4\lambda^2)^2} + \frac{f\pi^2}{32}\left\{\frac{1}{\lambda^2}\cos\frac{\pi y}{b} + \frac{1}{4\lambda^2}\cos\frac{2\pi y}{b} + \frac{\lambda^2\cos\frac{\pi x}{a}\cos\frac{\pi y}{b}}{(1+\lambda^2)^2}\right\}\right.$$

$$- \lambda^2\left\{a_1\cosh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\right\}\cos\frac{\pi y}{b}$$

$$\left. - 4\lambda^2\left\{a_2\cosh\frac{2\pi\lambda x}{a} + a_2\left(\frac{2\pi\lambda x}{a}\right)\sinh\frac{2\pi\lambda x}{a}\right\}\cos\frac{2\pi y}{b}\right]$$

$$\sigma_x\Big|_{x=a} = E\left(\frac{g}{R}\right)^2 \frac{f\lambda^2}{4}\left\{\left[\frac{f\pi^2}{32}\left\{\frac{1}{\lambda^2} - \frac{\lambda^2}{(1+\lambda^2)^2}\right\} - \lambda^2\left\{a_1\cosh\pi\lambda + b_1\pi\lambda\sinh\pi\lambda\right\}\right]\cos\frac{\pi y}{b}\right.$$

$$\left. + \left[\frac{f\pi^2}{32}\left\{\frac{1}{4\lambda^2}\right\} - 4\lambda^2\left\{a_2\cosh 2\pi\lambda + b_2 2\pi\lambda\sinh 2\pi\lambda\right\}\right]\cos\frac{2\pi y}{b}\right\}$$

Thus

$$\boxed{\begin{aligned} a_1 \cosh \pi\lambda + b_1 \pi\lambda \sinh \pi\lambda &= \frac{f\pi^2}{32}\left\{\frac{1}{\lambda^4} - \frac{1}{(1+\lambda^2)^2}\right\} \\ a_2 \cosh 2\pi\lambda + b_2 2\pi\lambda \sinh 2\pi\lambda &= \frac{f\pi^2}{32}\left\{\frac{1}{16\lambda^4}\right\} \end{aligned}}$$

$$\sigma_y = \frac{\partial^2 F}{\partial x^2} = E\left(\frac{a}{R}\right)^2 \frac{f\lambda^2}{4}\Big[-\frac{4\cos\frac{\pi x}{2a}\cos\frac{\pi y}{b}}{(1+4\lambda^2)^2} + \frac{f\pi^2}{32}\left\{\cos\frac{\pi x}{a} + \frac{\cos\frac{\pi x}{a}\cos\frac{\pi y}{b}}{(1+\lambda^2)^2}\right\} + 2a_0$$

$$+ \lambda^2\left\{(a_1+2b_1)\cosh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\right\}\cos\frac{\pi y}{b}$$

$$+ 4\lambda^2\left\{(a_2+2b_2)\cosh\frac{2\pi\lambda x}{a} + b_2\left(\frac{2\pi\lambda x}{a}\right)\sinh\frac{2\pi\lambda x}{a}\right\}\cos\frac{2\pi y}{b}\Big]$$

$$\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 = \left(\frac{a}{R}\right)^2 \frac{f\lambda^2}{4}\left[\frac{f\pi^2}{32} + \frac{f\pi^2}{32}\cos\frac{\pi x}{a} - \frac{f\pi^2}{32}\cos\frac{2\pi y}{b} - \frac{f\pi^2}{32}\cos\frac{\pi x}{a}\cos\frac{2\pi y}{b}\right]$$

$$\frac{\partial v}{\partial y} = \left(\frac{a}{R}\right)^2 \frac{f\lambda^2}{4}\Big[-\frac{4\cos\frac{\pi x}{2a}\cos\frac{\pi y}{b}}{(1+4\lambda^2)^2} + \frac{f\pi^2}{32}\left\{-1 + \cos\frac{2\pi y}{b} + \cos\frac{\pi x}{a}\cos\frac{2\pi y}{b} + \frac{\cos\frac{\pi x}{a}\cos\frac{\pi y}{b}}{(1+\lambda^2)^2}\right\}$$

$$+ 2a_0 + \lambda^2\left\{(a_1+2b_1)\cosh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\right\}\cos\frac{\pi y}{b}$$

$$+ 4\lambda^2\left\{(a_2+2b_2)\cosh\frac{2\pi\lambda x}{a} + b_2\left(\frac{2\pi\lambda x}{a}\right)\sinh\frac{2\pi\lambda x}{a}\right\}\cos\frac{2\pi y}{b}\Big]$$

$$\frac{v}{R} = \left(\frac{a}{R}\right)^3 \frac{f\lambda^2}{4}\left[-\frac{1}{\pi\lambda}\,\frac{4\cos\frac{\pi v}{2a}\sin\frac{\pi x}{b}}{(1+4\lambda^2)^2} + \left(\frac{f\pi^2}{32} + 9a_0'\right)\frac{1}{\lambda}\left(\frac{x}{b}\right) \quad + \cdots \quad + \cdots\cdots \right]$$

$$\therefore \quad \boxed{\left(\frac{v}{b}\right)_{\text{at } y=b} = \left(\frac{a}{R}\right)^2 \frac{f\lambda^2}{4}\left[-\frac{f\pi^2}{32} + 9a_0\right]}$$

$$\sigma_{y\,\text{at } y=b} = E\left(\frac{a}{R}\right)^2 \frac{f\lambda^2}{4}\left[+\frac{4\cos\frac{\pi v}{2a}}{(1+4\lambda^2)^2} + \frac{f\pi^2}{32}\left\{\cos\frac{\pi x}{a} - \frac{\cos\frac{\pi v}{a}}{(1+\lambda^2)^2}\right\} + 9a_0\right.$$

$$-\lambda^2\left\{(a_1+2b_1)\cosh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\right\}$$

$$\left. + 4\lambda^2\left\{(a_2+2b_2)\cosh\frac{2\pi\lambda x}{a} + b_2\left(\frac{2\pi\lambda x}{a}\right)\sinh\frac{2\pi\lambda x}{a}\right\}\right]$$

$$\boxed{\begin{aligned}-\frac{\sigma}{E} = \left(\frac{a}{R}\right)^2 \frac{f\lambda^2}{4}\Big[\, & \frac{8}{\pi}\,\frac{1}{(1+4\lambda^2)^2} - \frac{\lambda}{\pi}\left\{(a_1+b_1)\sinh\pi\lambda + b_1\,\pi\lambda\cosh\pi\lambda\right\} \\ & + 9a_0 \qquad + \frac{2\lambda}{\pi}\left\{(a_1+b_1)\sinh 2\pi\lambda + b_2\,2\pi\lambda\cosh 2\pi\lambda\right\}\Big]\end{aligned}}$$

$$\tau_{xy} = E\left(\frac{a}{R}\right)^2 \frac{f\lambda^2}{4}\left[-\frac{8\lambda \sin\frac{\pi x}{2a} \sin\frac{\pi y}{b}}{(1+4\lambda^2)^2} + \frac{f\pi^2}{32}\left\{\frac{\lambda \sin\frac{\pi x}{a} \sin\frac{\pi y}{b}}{(1+\lambda^2)^2}\right\}\right.$$

$$+\lambda^2\left\{(a_1+b_1)\sinh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\cosh\frac{\pi\lambda x}{a}\right\}\sin\frac{\pi y}{b}$$

$$\left.+4\lambda^2\left\{(a_2+b_2)\sinh\frac{2\pi\lambda x}{a} + b_2\left(\frac{2\pi\lambda x}{a}\right)\cosh\frac{2\pi\lambda x}{a}\right\}\sin\frac{2\pi y}{b}\right]$$

$$\therefore \boxed{\begin{aligned} &(a_2+b_2)\sinh 2\pi\lambda + b_2(2\pi\lambda)\cosh 2\pi\lambda = 0 \\ &(a_1+b_1)\sinh \pi\lambda + b_1(\pi\lambda)\cosh \pi\lambda = \frac{1}{\lambda}\frac{8}{(1+4\lambda^2)^2} \end{aligned}}$$

$$\therefore \quad -\frac{\sigma}{E} = \left(\frac{a}{R}\right)^2 \frac{f\lambda^2}{4} \qquad (9a_0), \qquad \left(\frac{v}{b}\right)_{at\, y=b} = -\frac{\sigma}{E} - \left(\frac{a}{R}\right)^2 \frac{f\lambda^2}{4}\frac{f\pi^2}{32}$$

$$\boxed{\frac{\Delta p}{Eabt} = -\left(\frac{\sigma}{E}\right)^2 - \left(\frac{\sigma}{E}\right)\left(\frac{a}{R}\right)^2\left(\frac{f}{4}\right)^2\left(\frac{\pi^2\lambda^2}{8}\right)}$$

$$\sigma_x = E\left(\frac{a}{R}\right)^2 \frac{f}{4}\left[\left(\frac{f\pi^2}{32} - \frac{16\lambda^4}{(1+4\lambda^2)^2}\cos\frac{\pi x}{2a} + \frac{f\pi^2}{32}\frac{\lambda^4}{(1+\lambda^2)^2}\cos\frac{\pi x}{a} - \lambda^4\left\{a_1\cosh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\right\}\right)\cos\frac{\pi y}{b}\right.$$

$$\left. + \left(\frac{f\pi^2}{128} - 4\lambda^4\left\{a_2\cosh\frac{2\pi\lambda x}{a} + b_1\left(\frac{2\pi\lambda x}{a}\right)\sinh\frac{2\pi\lambda x}{a}\right\}\right)\cos\frac{2\pi y}{b}\right]$$

$$\frac{1}{2ab}\int_0^a\int_0^b\left(\frac{\sigma_x}{E}\right)^2 dx\,dy = \left(\frac{a}{R}\right)^4\left(\frac{f}{4}\right)^2\left[\frac{1}{4}\left(\frac{f\pi^2}{32}\right)^2 + \frac{1}{8}\left\{\frac{16\lambda^4}{(1+4\lambda^2)^2}\right\}^2 + \frac{1}{8}\left\{\frac{f\pi^2}{32}\frac{\lambda^4}{(1+\lambda^2)^2}\right\}^2\right.$$

$$- \frac{1}{2}\frac{f\pi^2}{32}\lambda^4\int_0^1\left\{a_1\cosh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\right\}d\left(\frac{x}{a}\right)$$

$$+ \frac{8\lambda^8}{(1+4\lambda^2)^2}\int_0^1\cos\frac{\pi x}{2a}\left\{a_1\cosh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\right\}d\left(\frac{x}{a}\right)$$

$$- \frac{1}{2}\frac{f\pi^2}{32}\frac{\lambda^8}{(1+\lambda^2)^2}\int_0^1\cos\frac{\pi x}{a}\left\{a_1\cosh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\right\}d\left(\frac{x}{a}\right)$$

$$+ \frac{1}{4}\lambda^8\int_0^1\left\{a_1\cosh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\right\}^2 d\left(\frac{x}{a}\right) + \frac{1}{4}\left(\frac{f\pi^2}{128}\right)^2$$

$$- \frac{f\pi^2\lambda^4}{64}\int_0^1\left\{a_2\cosh\frac{2\pi\lambda x}{a} + b_2\left(\frac{2\pi\lambda x}{a}\right)\sinh\frac{2\pi\lambda x}{a}\right\}d\left(\frac{x}{a}\right)$$

$$\left. + 4\lambda^8\int_0^1\left\{a_2\cosh\frac{2\pi\lambda x}{a} + b_2\left(\frac{2\pi\lambda x}{a}\right)\sinh\frac{2\pi\lambda x}{a}\right\}^2 d\left(\frac{x}{a}\right)\right]$$

$$\sigma_y = E\left(\frac{g}{R}\right)^2\left(\frac{f}{4}\right)\Big[\frac{f\pi^2}{32}\lambda^2\cos\frac{\pi x}{a}$$

$$+\Big\{-\frac{4\lambda^2}{(1+4\lambda^2)^2}\cos\frac{\pi x}{2a}+\frac{f\pi^2}{32}\frac{\lambda^2}{(1+\lambda^2)^2}\cos\frac{\pi x}{a}+\lambda^4\Big[(a_1+2b_1)\cosh\frac{\pi\lambda x}{a}+b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\Big]\Big\}\cos\frac{\pi y}{b}$$

$$+\Big\{4\lambda^4\Big[(a_2+2b_2)\cosh\frac{2\pi\lambda x}{a}+b_2\left(\frac{2\pi\lambda x}{a}\right)\sinh\frac{2\pi\lambda x}{a}\Big]\Big\}\cos\frac{2\pi y}{b}\Big]-\frac{\sigma}{E}$$

$$\underline{\frac{1}{2ab}\int_0^a\int_0^b\left(\frac{\sigma_y}{E}\right)^2dx\,dy} = \left(\frac{g}{R}\right)^4\left(\frac{f}{4}\right)^2\Big[\frac{1}{4}\left(\frac{f\pi^2}{32}\lambda^2\right)^2+\frac{1}{8}\Big\{\frac{4\lambda^2}{(1+4\lambda^2)^2}\Big\}^2+\frac{1}{8}\Big\{\frac{f\pi^2}{32}\frac{\lambda^2}{(1+\lambda^2)^2}\Big\}^2$$

$$-\frac{2\lambda^6}{(1+4\lambda^2)^2}\int_0^1\cos\frac{\pi x}{2a}\Big\{(a_1+2b_1)\cosh\frac{\pi\lambda x}{a}+b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\Big\}d\left(\frac{x}{a}\right)$$

$$+\frac{1}{2}\frac{f\pi^2}{32}\frac{\lambda^6}{(1+\lambda^2)^2}\int_0^1\cos\frac{\pi x}{a}\Big\{(a_1+2b_1)\cosh\frac{\pi\lambda x}{a}+b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\Big\}d\left(\frac{x}{a}\right)$$

$$+\frac{\lambda^8}{4}\int_0^1\Big\{(a_1+2b_1)\cosh\frac{\pi\lambda x}{a}+b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\Big\}^2d\left(\frac{x}{a}\right)$$

$$+4\lambda^8\int_0^1\Big\{(a_2+2b_2)\cosh\frac{2\pi\lambda x}{a}+b_2\left(\frac{2\pi\lambda x}{a}\right)\sinh\frac{2\pi\lambda x}{a}\Big\}^2d\left(\frac{x}{a}\right)\Big]+\frac{1}{2}\left(\frac{\sigma}{E}\right)^2$$

$$\tau_{xy} = E\left(\frac{a}{R}\right)^2 \frac{f}{4}\left[\left\{-\frac{f\lambda^3}{(1+4\lambda^2)^2}\sin\frac{\pi x}{2a} + \frac{f\pi^2}{32}\frac{\lambda^3}{(1+\lambda^2)^2}\sin\frac{\pi x}{a} + \lambda^4\left[(a_1+b_1)\sinh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\cosh\frac{\pi\lambda x}{a}\right]\right\}\sin\frac{\pi y}{b}\right.$$

$$\left. + 4\lambda^4\left\{(a_2+b_2)\sinh\frac{2\pi\lambda x}{a} + b_2\left(\frac{2\pi\lambda x}{a}\right)\cosh\frac{2\pi\lambda x}{a}\right\}\sin\frac{2\pi y}{b}\right]$$

$$\frac{1}{ab}\int_0^a\int_0^b\left(\frac{\tau_{xy}}{E}\right)^2 dx\,dy = \left(\frac{a}{R}\right)^4\left(\frac{f}{4}\right)^2\left[\frac{1}{4}\left[\frac{f\lambda^3}{(1+4\lambda^2)^2}\right]^2 + \frac{1}{4}\left[\frac{f\pi^2}{32}\frac{\lambda^3}{(1+\lambda^2)^2}\right]^2\right.$$

$$- \frac{f\lambda^7}{(1+4\lambda^2)^2}\int_0^1 \sin\frac{\pi x}{2a}\left\{(a_1+b_1)\sinh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\cosh\frac{\pi\lambda x}{a}\right\}d\left(\frac{x}{a}\right)$$

$$+ \frac{f\pi^2}{32}\frac{\lambda^7}{(1+\lambda^2)^2}\int_0^1 \sin\frac{\pi x}{a}\left\{(a_1+b_1)\sinh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\cosh\frac{\pi\lambda x}{a}\right\}d\left(\frac{x}{a}\right)$$

$$+ \frac{1}{2}\lambda^8\int_0^1\left\{(a_1+b_1)\sinh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\cosh\frac{\pi\lambda x}{a}\right\}^2 d\left(\frac{x}{a}\right)$$

$$\left. + 8\lambda^8\int_0^1\left\{(a_2+b_2)\sinh\frac{2\pi\lambda x}{a} + b_2\left(\frac{2\pi\lambda x}{a}\right)\cosh\frac{2\pi\lambda x}{a}\right\}^2 d\left(\frac{x}{a}\right)\right]$$

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$$\int_0^1 \cosh\frac{\pi\lambda x}{a}\, d\left(\frac{x}{a}\right) = \frac{1}{\pi\lambda}\sinh\pi\lambda$$

$$\int_0^1 \frac{\pi\lambda x}{a}\sinh\frac{\pi\lambda x}{a}\, d\left(\frac{x}{a}\right) = \frac{1}{\pi\lambda}\int_0^{\pi\lambda}\theta\sinh\theta\, d\theta = \frac{1}{\pi\lambda}\left[\pi\lambda\cosh\pi\lambda - \sinh\pi\lambda\right] = \cosh\pi\lambda - \frac{\sinh\pi\lambda}{\pi\lambda}$$

$$\int_0^1 \cos\frac{\pi x}{2a}\cosh\frac{\pi\lambda x}{a}\, d\left(\frac{x}{a}\right) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left[\cosh(2\lambda+i)\theta + \cosh(2\lambda-i)\theta\right]d\theta$$

$$= \frac{1}{\pi}\left[\frac{\sinh(2\lambda+i)\frac{\pi}{2}}{2\lambda+i} + \frac{\sinh(2\lambda-i)\frac{\pi}{2}}{2\lambda-i}\right] = \frac{2}{\pi}\frac{\cosh\pi\lambda}{(1+4\lambda^2)}$$

$$\int_0^1 \cos\frac{\pi x}{2a}\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\, d\left(\frac{x}{a}\right) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} 2\lambda\theta\left[\sinh(2\lambda+i)\theta + \sinh(2\lambda-i)\theta\right]d\theta$$

$$= \frac{2\lambda}{\pi}\left[\left\{\frac{\cosh(2\lambda+i)\frac{\pi}{2}}{2\lambda+i} + \frac{\cosh(2\lambda-i)\frac{\pi}{2}}{2\lambda-i}\right\}\frac{\pi}{2} - \left\{\frac{\sinh(2\lambda+i)\frac{\pi}{2}}{(2\lambda+i)^2} + \frac{\sinh(2\lambda-i)\frac{\pi}{2}}{(2\lambda-i)^2}\right\}\right]$$

$$= \frac{2\lambda}{\pi}\left[\frac{\pi\sinh\pi\lambda}{1+4\lambda^2} - \frac{8\lambda\cosh\pi x}{(1+4\lambda^2)^2}\right] = \left[\frac{2\lambda\sinh\pi\lambda}{(1+4\lambda^2)} - \frac{16\lambda^2\cosh\pi\lambda}{\pi(1+4\lambda^2)^2}\right]$$

$$\int_0^1 \cos\frac{\pi x}{a}\cosh\frac{\pi\lambda x}{a}\, d\left(\frac{x}{a}\right) = \frac{1}{2\pi}\int_0^{\pi}\left[\cosh(\lambda+i)\theta + \cosh(\lambda-i)\theta\right]d\theta$$

$$= \frac{1}{2\pi}\left[\frac{\sinh(\lambda+i)\pi}{\lambda+i} + \frac{\sinh(\lambda-i)\pi}{\lambda-i}\right] = -\frac{\lambda}{\pi}\frac{\sinh\lambda\pi}{1+\lambda^2}$$

$$\int_0^1 \cos\frac{\pi x}{a}\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\, d\left(\frac{x}{a}\right) = \frac{1}{2\pi}\int_0^{\pi}\lambda\theta\left[\sinh(\lambda+i)\theta + \sinh(\lambda-i)\theta\right]d\theta$$

$$= \frac{\lambda}{2\pi}\left[\pi\left\{\frac{\cosh(\lambda+i)\pi}{\lambda+i} + \frac{\cosh(\lambda-i)\pi}{\lambda-i}\right\} - \left\{\frac{\sinh(\lambda+i)\pi}{(\lambda+i)^2} + \frac{\sinh(\lambda-i)\pi}{(\lambda-i)^2}\right\}\right]$$

$$= \frac{\lambda}{2\pi}\left[-\frac{2\lambda\pi\cosh\lambda\pi}{1+\lambda^2} - \frac{2(1-\lambda^2)\sinh\lambda\pi}{(1+\lambda^2)^2}\right]$$

$$\int_0^1 \cosh^2\frac{\pi\lambda x}{a}\, d\left(\frac{x}{a}\right) = \frac{1}{2}\int_0^1\left[1+\cosh\frac{2\pi\lambda x}{a}\right]d\left(\frac{x}{a}\right) = \frac{1}{2} + \frac{\sinh 2\pi\lambda}{2\pi\lambda}$$

$$\int_0^1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\cosh\frac{\pi\lambda x}{a}\, d\left(\frac{x}{a}\right) = \frac{1}{2}\int_0^1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{2\pi\lambda x}{a}\, d\left(\frac{x}{a}\right) = \frac{1}{8\pi\lambda}\left[2\pi\lambda\cosh 2\pi\lambda - \sinh 2\pi\lambda\right]$$

$$= \frac{1}{4}\left[\cosh 2\pi\lambda - \frac{\sinh 2\pi\lambda}{2\pi\lambda}\right]$$

$$\int_0^1\left(\frac{\pi\lambda x}{a}\right)^2\sinh^2\frac{\pi\lambda x}{a}\, d\left(\frac{x}{a}\right) = \frac{1}{2}\int_0^1\left(\frac{\pi\lambda x}{a}\right)^2\left[\cosh\frac{2\pi\lambda x}{a} - 1\right]d\left(\frac{x}{a}\right) = \frac{1}{2}\left[-\frac{1}{3}(\pi\lambda)^2\right.$$

$$\left. + \frac{1}{8\pi\lambda}\left\{(x^2+2)\sinh x - 2x\cosh x\right\}_0^{2\pi\lambda}\right] = \frac{1}{2}\left[-\frac{(\pi\lambda)^2}{3} + \frac{1}{8\pi\lambda}\left\{(4\pi^2\lambda^2+2)\sinh 2\pi\lambda - 4\pi\lambda\cosh 2\pi\lambda\right\}\right]$$

$$= -\frac{(\pi\lambda)^2}{6} + \frac{1}{8}\left[(4\pi^2\lambda^2+2)\frac{\sinh 2\pi\lambda}{2\pi\lambda} - 2\cosh 2\pi\lambda\right]$$

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$$\int_0^1 \sin\frac{\pi x}{2a}\,\sinh\frac{\pi\lambda x}{a}\,d\left(\frac{x}{a}\right) = \frac{1}{\pi i}\int_0^{\frac{\pi}{2}}\left[\cosh(2\lambda+i)\theta - \cosh(2\lambda-i)\theta\right]d\theta$$

$$= \frac{1}{\pi i}\left[\frac{\sinh(2\lambda+i)\theta}{2\lambda+i} - \frac{\sinh(2\lambda-i)\theta}{2\lambda-i}\right] = \frac{4\lambda}{\pi}\,\frac{\cosh\pi\lambda}{1+4\lambda^2}$$

$$\int_0^1 \sin\frac{\pi x}{2a}\left(\frac{\pi\lambda x}{a}\right)\cosh\frac{\pi\lambda x}{a}\,d\left(\frac{x}{a}\right) = \frac{1}{\pi i}\,2\lambda\int_0^{\frac{\pi}{2}}\theta\left[\sinh(2\lambda+i)\theta - \sinh(2\lambda-i)\theta\right]d\theta$$

$$= \frac{2\lambda}{\pi i}\left[\frac{\pi}{2}\left\{\frac{\cosh(2\lambda+i)\frac{\pi}{2}}{2\lambda+i} \quad \frac{\cosh(2\lambda+i)\frac{\pi}{2}}{2\lambda-i}\right\} - \left\{\frac{\sinh(2\lambda+i)\frac{\pi}{2}}{(2\lambda+i)^2} - \frac{\sinh(2\lambda-i)\frac{\pi}{2}}{(2\lambda+i)^2}\right\}\right]$$

$$= \frac{2\lambda}{\pi}\left[\frac{2\pi\lambda\sinh\pi\lambda}{(1+4\lambda^2)} + \frac{2(1-4\lambda^2)\cosh\pi\lambda}{(1+4\lambda^2)^2}\right]$$

$$\int_0^1 \sin\frac{\pi x}{a}\,\sinh\frac{\pi\lambda x}{a}\,d\left(\frac{x}{a}\right) = \frac{1}{2\pi i}\int_0^{\pi}\left[\cosh(\lambda+i)\theta - \cosh(\lambda-i)\theta\right]d\theta$$

$$= \frac{1}{2\pi i}\left[\frac{\sinh(\lambda+i)\pi}{\lambda+i} - \frac{\sinh(\lambda-i)\pi}{\lambda-i}\right] = \frac{1}{\pi}\,\frac{\sinh\lambda\pi}{(1+\lambda^2)}$$

$$\int_0^1 \sin\frac{\pi x}{a}\left(\frac{\pi\lambda x}{a}\right)\cosh\frac{\pi\lambda x}{a}\,d\left(\frac{x}{a}\right) = \frac{\lambda}{2\pi i}\int_0^{\pi}\theta\left[\sinh(\lambda+i)\theta - \sinh(\lambda-i)\theta\right]d\theta$$

$$= \frac{\lambda}{2\pi i}\left[\pi\left\{\frac{\cosh(\lambda+i)\pi}{\lambda+i} - \frac{\cosh(\lambda-i)\pi}{\lambda-i}\right\} - \left\{\frac{\sinh(\lambda+i)\pi}{(\lambda+i)^2} - \frac{\sinh(\lambda-i)\pi}{(\lambda-i)^2}\right\}\right]$$

$$= \frac{\lambda}{2\pi}\left[\frac{2\pi\cosh\lambda\pi}{1+\lambda^2} - \frac{4\lambda\sinh\lambda\pi}{(1+\lambda^2)^2}\right]$$

$$\int_0^1 \sinh^2 \frac{\pi\lambda x}{a}\, d\left(\frac{x}{a}\right) = \frac{1}{2}\int_0^1 \left[\cosh \frac{2\pi\lambda x}{a} - \frac{1}{2}\right] d\left(\frac{x}{a}\right) = -\frac{1}{2} + \frac{\sinh 2\pi\lambda}{2\pi\lambda}$$

$$\int_0^1 \left(\frac{\pi\lambda x}{a}\right)^2 \cosh^2 \frac{\pi\lambda x}{a}\, d\left(\frac{x}{a}\right) = \frac{1}{2}\int_0^1 \left(\frac{\pi\lambda x}{a}\right)^2 \left[1 + \cosh \frac{2\pi x\lambda}{a}\right] d\left(\frac{x}{a}\right)$$

$$= \frac{1}{6}(\pi\lambda)^2 + \frac{1}{8}\left[(4\pi^2\lambda^2 \cdot 2)\frac{\sinh 2\pi\lambda}{2\pi\lambda} - 2\cosh 2\pi\lambda\right]$$

5/0

$$
\begin{aligned}
\frac{1}{2ab}\int_0^a\int_0^b \left(\frac{\sigma_x}{E}\right)^2 dx\,dy = \left(\frac{a}{R}\right)^4\left(\frac{f}{4}\right)^2 \Bigg[& \frac{1}{4}\left(\frac{f\pi^2}{32}\right)^2 + \frac{1}{8}\frac{256\lambda^8}{(1+4\lambda^2)^4} + \frac{1}{8}\frac{\lambda^8}{(1+\lambda^2)^4}\left(\frac{f\pi^2}{32}\right)^2 \\
& - \frac{f}{2}\frac{f\pi^2}{32}\lambda^4\left[(a_1-b_1)\frac{\sinh\pi\lambda}{\pi\lambda} + b_1\cosh\pi\lambda\right] \\
& + \frac{f\lambda^8}{(1+4\lambda^2)^2}\left[\frac{2}{\pi}\frac{\cosh\pi\lambda}{(1+4\lambda^2)}a_1 + \left\{\frac{2\lambda\sinh\pi\lambda}{(1+4\lambda^2)} - \frac{16\lambda^2\cosh\pi\lambda}{\pi(1+4\lambda^2)^2}\right\}b_1\right] \\
& + \frac{f\pi^2}{64}\frac{\lambda^8}{(1+\lambda^2)^2}\left[+\frac{\lambda}{\pi}\frac{\sinh\lambda\pi}{(1+\lambda^2)}a_1 + \left\{-\frac{\lambda^2\cosh\lambda\pi}{(1+\lambda^2)} + \frac{\lambda(1-\lambda^2)\sinh\lambda\pi}{\pi(1+\lambda^2)^2}\right\}b_1\right] \\
& + \frac{\lambda^8}{4}\left[a_1^2\left\{\frac{1}{2} + \frac{\sinh 2\pi\lambda}{2\pi\lambda}\right\} + 2a_1b_1\frac{1}{4}\left\{\cosh 2\pi\lambda - \frac{\sinh 2\pi\lambda}{2\pi\lambda}\right\} + b_1^2\left\{-\frac{\pi^2\lambda^2}{6} + \frac{1}{8}\left[(4\pi^2\lambda^2+2)\frac{\sinh 2\pi\lambda}{2\pi\lambda} - 2\cosh 2\pi\lambda\right]\right\}\right] \\
& + \frac{1}{4}\left(\frac{f\pi^2}{128}\right)^2 \\
& - \frac{f\pi^2}{64}\lambda^4\left[(a_2-b_2)\frac{\sinh 2\pi\lambda}{2\pi\lambda} + b_2\cosh 2\pi\lambda\right] \\
& + 4\lambda^8\left[a_2^2\left\{\frac{1}{2} + \frac{\sinh 4\pi\lambda}{4\pi\lambda}\right\} + 2a_2b_2\frac{1}{4}\left\{\cosh 4\pi\lambda - \frac{\sinh 4\pi\lambda}{4\pi\lambda}\right\} + b_2^2\left\{-\frac{4\pi^2\lambda^2}{6} + \frac{1}{8}\left[(16\pi^2\lambda^2+2)\frac{\sinh 4\pi\lambda}{4\pi\lambda} - 2\cosh 4\pi\lambda\right]\right\}\right]\Bigg]
\end{aligned}
$$

$$\frac{1}{2ab}\int_0^a\int_0^b\left(\frac{\sigma_y}{E}\right)^2 dx\,dy = \left(\frac{a}{R}\right)^4\left(\frac{f}{4}\right)^2\Bigg[\frac{\lambda^4}{4}\left(\frac{f\pi^2}{32}\right)^2 + \frac{1}{8}\,\frac{16\lambda^4}{(1+4\lambda^2)^4} + \frac{1}{8}\,\frac{\lambda^4}{(1+\lambda^2)^4}\left(\frac{f\pi^2}{32}\right)^2$$

$$-\frac{2\lambda^6}{(1+4\lambda^2)^2}\left\{\frac{2}{\pi}\,\frac{\cosh\pi\lambda}{(1+4\lambda^2)}(a_1+2b_1) + \left[\frac{2\lambda\sinh\pi\lambda}{(1+4\lambda^2)} - \frac{16\lambda^2\cosh\pi\lambda}{\pi(1+4\lambda^2)^2}\right]b_1\right\}$$

$$-\frac{f\pi^2}{64}\,\frac{\lambda^6}{(1+\lambda^2)^2}\left\{\frac{\lambda}{\pi}\,\frac{\sinh\lambda\pi}{(1+\lambda^2)}(a_1+2b_1) + \left[\frac{\lambda^2\cosh\lambda\pi}{(1+\lambda^2)} + \frac{\lambda(1-\lambda^2)\sinh\lambda\pi}{\pi(1+\lambda^2)^2}\right]b_1\right\}$$

$$+\frac{\lambda^8}{4}\Bigg\{(a_1+2b_1)^2\left[\frac{1}{2}+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right] + 2b_1(a_1+2b_1)\frac{1}{4}\left[\cosh 2\pi\lambda - \frac{\sinh 2\pi\lambda}{2\pi\lambda}\right]$$

$$+ b_1^2\left[-\frac{\pi^2\lambda^2}{6} + \frac{1}{8}\left[(4\pi^2\lambda^2+2)\frac{\sinh 2\pi\lambda}{2\pi\lambda} - 2\cosh 2\pi\lambda\right]\right]\Bigg\}$$

$$+4\lambda^8\Bigg\{(a_2+2b_2)^2\left[\frac{1}{2}+\frac{\sinh 4\pi\lambda}{4\pi\lambda}\right] + 2b_2(a_2+2b_2)\frac{1}{4}\left[\cosh 4\pi\lambda - \frac{\sinh 4\pi\lambda}{4\pi\lambda}\right]$$

$$+b_2^2\left[-\frac{4\pi^2\lambda^2}{6} + \frac{1}{8}\left[(16\pi^2\lambda^2+2)\frac{\sinh 4\pi\lambda}{4\pi\lambda} - 2\cosh 4\pi\lambda\right]\right]\Bigg\}\Bigg] + \frac{1}{2}\left(\frac{\sigma}{E}\right)^2$$

$$\frac{1}{ab}\int_0^a\int_0^b \left(\frac{\tau_{xy}}{E}\right)^2 dx\,dy = \left(\frac{g}{R}\right)^4\left(\frac{f}{4}\right)^2\Bigg[\frac{1}{4}\frac{64\lambda^6}{(1+4\lambda^2)^4} + \frac{1}{4}\frac{\lambda^6}{(1+\lambda^2)^4}\left(\frac{f\pi^2}{32}\right)^2$$

$$-\frac{8\lambda^7}{(1+4\lambda^2)^2}\left\{\frac{4\lambda}{\pi}\frac{\cosh\pi\lambda}{1+4\lambda^2}(a_1+b_1) + \left[\frac{4\lambda^2\sinh\pi\lambda}{(1+4\lambda^2)} + \frac{4\lambda(1-4\lambda^2)\cosh\pi\lambda}{\pi(1+4\lambda^2)^2}\right]b_1\right\}$$

$$+\frac{f\pi^2}{32}\frac{\lambda^7}{(1+\lambda^2)^2}\left\{\frac{1}{\pi}\frac{\sinh\lambda\pi}{(1+\lambda^2)}(a_1+b_1) + \left[\frac{\lambda\cosh\lambda\pi}{(1+\lambda^2)} - \frac{2\lambda^2\sinh\lambda\pi}{\pi(1+\lambda^2)^2}\right]b_1\right\}$$

$$+\frac{\lambda^8}{2}\Bigg\{(a_1+b_1)^2\left[-\frac{1}{2}+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right] + 2b_1(a_1+b_1)\frac{1}{4}\left[\cosh 2\pi\lambda - \frac{\sinh 2\pi\lambda}{2\pi\lambda}\right]$$

$$+ b_1^2\left[\frac{\pi^2\lambda^2}{6} + \frac{1}{8}\left[(4\pi^2\lambda^2+2)\frac{\sinh 2\pi\lambda}{2\pi\lambda} - 2\cosh 2\pi\lambda\right]\right]\Bigg\}$$

$$+8\lambda^8\Bigg\{(a_2+b_2)^2\left[-\frac{1}{2}+\frac{\sinh 4\pi\lambda}{4\pi\lambda}\right] + 2b_2(a_2+b_2)\frac{1}{4}\left[\cosh 4\pi\lambda - \frac{\sinh 4\pi\lambda}{4\pi\lambda}\right]$$

$$+b_2^2\left[\frac{4\pi^2\lambda^2}{6} + \frac{1}{8}\left[(16\pi^2\lambda^2+2)\frac{\sinh 4\pi\lambda}{4\pi\lambda} - 2\cosh 4\pi\lambda\right]\right]\Bigg\}\Bigg]$$

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$$\cosh \pi\lambda \cdot a_1 + \pi\lambda \sinh \pi\lambda \cdot b_1 = \frac{f\pi^2}{32}\left\{\frac{1}{\lambda^4} - \frac{1}{(1+\lambda^2)^2}\right\}$$

$$\sinh \pi\lambda \cdot a_1 + (\sinh \pi\lambda + \pi\lambda \cosh \pi\lambda)\, b_1 = \frac{1}{\lambda}\frac{8}{(1+4\lambda^2)^2}$$

$$(\pi\lambda + \cosh \pi\lambda \cdot \sinh \pi\lambda)\, b_1 = \frac{1}{\lambda}\frac{8}{(1+4\lambda^2)^2}\cosh \pi\lambda - \frac{f\pi^2}{32}\left\{\frac{1}{\lambda^4} - \frac{1}{(1+\lambda^2)^2}\right\}\sinh \pi\lambda$$

$$\boxed{b_1 = \frac{\dfrac{8}{\lambda(1+4\lambda^2)^2}\dfrac{\cosh \pi\lambda}{\pi\lambda} - \dfrac{f\pi^2}{32}\left\{\dfrac{1}{\lambda^4} - \dfrac{1}{(1+\lambda^2)^2}\right\}\dfrac{\sinh \pi\lambda}{\pi\lambda}}{1 + \dfrac{\sinh 2\pi\lambda}{2\pi\lambda}}}$$

$$a_1 = \frac{8}{\lambda(1+4\lambda^2)^2}\frac{1}{\sinh \pi\lambda} - \frac{\sinh \pi\lambda + \pi\lambda \cosh \pi\lambda}{\sinh \pi\lambda}\, b_1$$

$$= \frac{8}{\lambda(1+4\lambda^2)^2}\frac{1}{\sinh \pi\lambda} - \frac{(\sinh \pi\lambda + \pi\lambda \cosh \pi\lambda)\left[\dfrac{8}{\lambda(1+4\lambda^2)^2}\cosh \pi\lambda - \dfrac{f\pi^2}{32}\left\{\dfrac{1}{\lambda^4} - \dfrac{1}{(1+\lambda^2)^2}\right\}\sinh \pi\lambda\right]}{\sinh \pi\lambda\,(\pi\lambda + \sinh \pi\lambda \cosh \pi\lambda)}$$

$$= \frac{-\dfrac{8\pi\lambda}{\lambda(1+4\lambda^2)^2}\sinh^2 \pi\lambda + (\sinh \pi\lambda + \pi\lambda \cosh \pi\lambda)\sinh \pi\lambda\,\dfrac{f\pi^2}{32}\left\{\dfrac{1}{\lambda^4} - \dfrac{1}{(1+\lambda^2)^2}\right\}}{\sinh \pi\lambda\,(\pi\lambda + \sinh \pi\lambda \cosh \pi\lambda)}$$

$$\boxed{a_1 = \frac{-\dfrac{8}{\lambda(1+4\lambda^2)^2}\sinh \pi\lambda + \left(\dfrac{\sinh \pi\lambda}{\pi\lambda} + \cosh \pi\lambda\right)\dfrac{f\pi^2}{32}\left\{\dfrac{1}{\lambda^4} - \dfrac{1}{(1+\lambda^2)^2}\right\}}{1 + \dfrac{\sinh 2\pi\lambda}{2\pi\lambda}}}$$

$$\cosh 2\pi\lambda \cdot a_2 + 2\pi\lambda \sinh 2\pi\lambda \; b_2 = \frac{1}{\lambda^4}\frac{f\pi^2}{512}$$

$$\sinh 2\pi\lambda \, a_2 + (\sinh 2\pi\lambda + 2\pi\lambda \cosh 2\pi\lambda) \, b_2 = 0$$

$$\boxed{b_2 = - \frac{\frac{1}{\lambda^4}\frac{f\pi^2}{512} \cdot \frac{\sinh 2\pi\lambda}{2\pi\lambda}}{1 + \frac{\sinh 4\pi\lambda}{4\pi\lambda}}}$$

$$a_2 = - \frac{\frac{\sinh 2\pi\lambda}{2\pi\lambda} + \cosh 2\pi\lambda}{\frac{\sinh 2\pi\lambda}{2\pi\lambda}} b_2$$

$$\boxed{a_2 = + \frac{\frac{1}{\lambda^4}\frac{f\pi^2}{512} \cdot \left(\cosh 2\pi\lambda + \frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)}{1 + \frac{\sinh 4\pi\lambda}{4\pi\lambda}}}$$

Terms depends upon a_1, b_1, lineraly

$$- \frac{f\pi^2}{64}\lambda^4\left[(a_1-b_1)\frac{\sinh \pi\lambda}{\pi\lambda} + b_1 \cosh \pi\lambda\right]$$

$$+ \frac{1}{\pi}\frac{\cosh \pi\lambda}{(1+4\lambda^2)^3}\left\{16\lambda^8 a_1 - 4\lambda^6(a_1+2b_1) - 32\lambda^8(a_1+b_1)\right\}$$

$$+ \left\{\frac{2\lambda \sinh \pi\lambda}{(1+4\lambda^2)^3} - \frac{16\lambda^2 \cosh \pi\lambda}{\pi(1+4\lambda^2)^4}\right\}\left\{8\lambda^8 b_1 - 2\lambda^6 b_1\right\}$$

$$- \left\{\frac{2\lambda \sinh \pi\lambda}{(1+4\lambda^2)^3} + \frac{2(1-4\lambda^2)\cosh \pi\lambda}{\pi(1+4\lambda^2)^4}\right\} 16\lambda^8 b_1$$

$$+ \frac{f\pi^2}{64}\frac{\lambda}{\pi}\frac{\sinh \lambda\pi}{(1+\lambda^2)^3}\left\{\lambda^8 a_1 - \lambda^6(a_1+2b_1) + 2\lambda^6(a_1+b_1)\right\}$$

$$+ \frac{f\pi^2}{64}\left[\left\{\frac{\lambda^2\cosh \lambda\pi}{(1+\lambda^2)^3} + \frac{\lambda(1-\lambda^2)\sinh \lambda\pi}{\pi(1+\lambda^2)^4}\right\}\left\{\lambda^8 b_1 - \lambda^6 b_1\right\} + \left\{\frac{\lambda^2 \cosh \lambda\pi}{(1+\lambda^2)^3} - \frac{2\lambda^3 \sinh \lambda\pi}{\pi(1+\lambda^2)^4}\right\} 2\lambda^6 b_1\right]$$

$$- \frac{f\pi^2}{64}\lambda^4\left[(a_1-b_1)\frac{\sinh \pi\lambda}{\pi\lambda} + b_1 \cosh \pi\lambda\right]$$

$$- \frac{4\lambda^6}{\pi}\frac{\cosh \pi\lambda}{(1+4\lambda^2)^2}(a_1+2b_1)$$

$$- \frac{4\lambda^7 \sinh \pi\lambda}{(1+4\lambda^2)^2} b_1$$

$$+ \frac{f\pi^2}{64}\frac{\lambda^7 \sinh \lambda\pi}{\pi(1+\lambda^2)^2} a_1$$

$$+ \frac{f\pi^2}{64}\left[\frac{\lambda^8 \cosh \pi\lambda}{(1+\lambda^2)^2} b_1 - \frac{\lambda^7 \sinh \lambda\pi}{\pi(1+\lambda^2)^2} b_1\right]$$

Terms depends linearly upon a_2, b_2

$$- \frac{f\pi^2}{64}\lambda^4\left[(a_2-b_2)\frac{\sinh 2\pi\lambda}{2\pi\lambda} + b_2 \cosh 2\pi\lambda\right]$$

Terms in second order of a_1, b_1

$$\frac{\lambda^8}{4}\Big[\left\{\frac{1}{2}+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right\}(a_1^2+a_1^2+4a_1b_1+4b_1^2)+\frac{1}{2}\left\{\cosh 2\pi\lambda-\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right\}(a_1b_1+a_1b_1+2b_1^2)$$

$$+\left\{-\frac{\pi^2\lambda^2}{6}+\frac{1}{8}\left[(4\pi^2\lambda^2+2)\frac{\sinh 2\pi\lambda}{2\pi\lambda}-2\cosh 2\pi\lambda\right]\right\}(b_1^2+b_1^2)$$

$$+\left\{-\frac{1}{2}+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right\}(2a_1^2+4a_1b_1+2b_1^2)+\frac{1}{2}\left\{\cosh 2\pi\lambda-\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right\}(2a_1b_1+2b_1^2)$$

$$+\left\{\frac{\pi^2\lambda^2}{6}+\frac{1}{8}\left[(4\pi^2\lambda^2+2)\frac{\sinh 2\pi\lambda}{2\pi\lambda}-2\cosh 2\pi\lambda\right]\right\}(2b_1^2)\Big]$$

$$=\frac{\lambda^8}{4}\Big[4\left(\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)(a_1+b_1)^2+2b_1^2\left(\frac{1}{2}+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)+2\left\{\cosh 2\pi\lambda-\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right\}(a_1b_1+b_1^2)$$

$$+\frac{1}{2}b_1^2\left\{(4\pi^2\lambda^2+2)\frac{\sinh 2\pi\lambda}{2\pi\lambda}-2\cosh 2\pi\lambda\right\}\Big]$$

$$=\frac{\lambda^8}{4}\Big[4\left(\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)a_1^2+\left\{3\left(\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)+\cosh 2\pi\lambda\right\}2a_1b_1+\left\{1+(2\pi^2\lambda^2+5)\frac{\sinh 2\pi\lambda}{2\pi\lambda}+\cosh 2\pi\lambda\right\}b_1^2\Big]$$

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Terms in second order of a_2, b_2

$$4\lambda^8\left[4\left(\frac{\sinh 4\pi\lambda}{4\pi\lambda}\right)^{-}a_2^2+\left\{3\left(\frac{\sinh 4\pi\lambda}{4\pi\lambda}\right)+\cosh 4\pi\lambda\right\}2a_2b_2+\left\{1+(8\pi^2\lambda^2+5)\frac{\sinh 4\pi\lambda}{4\pi\lambda}+\cosh 4\pi\lambda\right\}b_2^2\right]$$

Terms independ of a_1, b_1, a_2, b_2

$$\left\{\frac{17}{4(128)^2}+\frac{\lambda^4}{4(32)^2}+\frac{1}{8(32)^2}\frac{\lambda^4}{(1+\lambda^2)^2}\right\}(f\pi^2)^2+\frac{2\lambda^4}{(1+4\lambda^2)^2}$$

$$-\frac{f\pi^2}{64}\lambda^4\left[(a_2-b_2)\frac{\sinh 2\pi\lambda}{2\pi\lambda}+b_2\cosh 2\pi\lambda\right]=-\frac{(f\pi^2)^2}{64\times 512}\left[\frac{\left(\cosh 2\pi\lambda+\frac{2\sinh 2\pi\lambda}{2\pi\lambda}\right)\frac{\sinh 2\pi\lambda}{2\pi\lambda}-\frac{\sinh 2\pi\lambda}{2\pi\lambda}\cosh 2\pi\lambda}{1+\frac{\sinh 4\pi\lambda}{4\pi\lambda}}\right]$$

$$=-\frac{(f\pi^2)^2}{32\times 512}\frac{\left(\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2}{1+\frac{\sinh 4\pi\lambda}{4\pi\lambda}}$$

$$4\lambda^8\left[4\left(\frac{\sinh 4\pi\lambda}{4\pi\lambda}\right)a_2^2 + \left\{3\left(\frac{\sinh 4\pi\lambda}{4\pi\lambda}\right)+\cosh 4\pi\lambda\right\}2a_2b_2 + \left\{1+(8\pi^2\lambda^2+5)\frac{\sinh 4\pi\lambda}{4\pi\lambda}+\cosh 4\pi\lambda\right\}b_2^2\right]$$

$$\begin{aligned}
= \frac{\left(\frac{f\pi^2}{256}\right)^2\frac{\sinh 2\pi\lambda}{2\pi\lambda}}{\left(1+\frac{\sinh 4\pi\lambda}{4\pi\lambda}\right)^2}\Bigg[\, & 4\cosh 2\pi\lambda\left\{\cosh 2\pi\lambda+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right\}^2 - 2\left\{3\left(\frac{\sinh 4\pi\lambda}{4\pi\lambda}\right)+\cosh 4\pi\lambda\right\}\left\{\cosh 2\pi\lambda+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right\} \\
& + \frac{\sinh 2\pi\lambda}{2\pi\lambda}\left\{1+(8\pi^2\lambda^2+5)\frac{\sinh 4\pi\lambda}{4\pi\lambda}+\cosh 4\pi\lambda\right\}\Bigg]
\end{aligned}$$

$$\begin{aligned}
= \frac{\left(\frac{f\pi^2}{256}\right)^2\frac{\sinh 2\pi\lambda}{2\pi\lambda}}{\left(1+\frac{\sinh 4\pi\lambda}{4\pi\lambda}\right)^2}\Bigg[\, & 4\cosh 2\pi\lambda\left\{\frac{1}{2}(\cosh 4\pi\lambda+1)+\cancel{2\frac{\sinh 4\pi\lambda}{4\pi\lambda}}+\cancel{\left(\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2}\right\} \\
& +6_2\frac{\sinh 4\pi\lambda}{4\pi\lambda}\cdot\cosh 2\pi\lambda - \cancel{2\cosh 4\pi\lambda\cosh 2\pi\lambda} - \cancel{6\frac{\sinh 4\pi\lambda}{4\pi\lambda}\frac{\sinh 2\pi\lambda}{2\pi\lambda}} \\
& - 2\cosh 4\pi\lambda\frac{\sinh 2\pi\lambda}{2\pi\lambda} \\
& + \frac{\sinh 2\pi\lambda}{2\pi\lambda} + (8\pi^2\lambda^2+\overset{3}{5})\frac{\sinh 2\pi\lambda}{2\pi\lambda}\frac{\sinh 4\pi\lambda}{4\pi\lambda} + \cancel{\frac{\sinh 2\pi\lambda}{2\pi\lambda}\cosh 4\pi\lambda}\Bigg]
\end{aligned}$$

$$= \frac{\left(\frac{f\pi^2}{256}\right)^2\left(\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)}{\left(1+\frac{\sinh 4\pi\lambda}{4\pi\lambda}\right)^2}\left\{(8\pi^2\lambda^2+3)\frac{\sinh 4\pi\lambda}{4\pi\lambda}\frac{\sinh 2\pi\lambda}{2\pi\lambda}+2\left(-\cosh 2\pi\lambda+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)\right\}$$

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The terms quadratic in a_1, b_1 can be divided into three parts:

$$\frac{\left(\frac{f\pi^2}{64}\right)^2\left\{1-\frac{\lambda^4}{(1+\lambda^2)^2}\right\}^2\left(\frac{\sinh\pi\lambda}{\pi\lambda}\right)}{\left(1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2}\left\{(2\lambda^2\pi^2+3)\frac{\sinh\pi\lambda}{\pi\lambda}\frac{\sinh 2\pi\lambda}{2\pi\lambda}+2\left(\cosh\pi\lambda+\frac{\sinh\pi\lambda}{\pi\lambda}\right)\right\}$$

$$\frac{\left(\frac{f\pi^2}{16}\right)\left\{1-\frac{\lambda^4}{(1+\lambda^2)^2}\right\}\frac{\lambda^4}{\lambda(1+4\lambda^2)^2}\frac{1}{\pi\lambda}}{\left(1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2}\left[\frac{2}{4}\left(\frac{\sinh 2\pi\lambda}{\quad}\right)\left\{-2\sinh\pi\lambda\left(\frac{\sinh\pi\lambda}{\pi\lambda}+\cosh\pi\lambda\right)\right\}\right.$$

$$+2\left\{3\left(\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)+\cosh 2\pi\lambda\right\}\left\{\sinh\pi\lambda\frac{\sinh\pi\lambda}{\quad}+\frac{\sinh 2\pi\lambda}{2\pi\lambda}+\cosh\pi\lambda\cosh\pi\lambda\right\}$$

$$\left.-2\left\{1+(2\pi^2\lambda^2+5)\frac{\sinh 2\pi\lambda}{2\pi\lambda}+\cosh 2\pi\lambda\right\}\left\{\frac{\sinh\pi\lambda}{\pi\lambda}\frac{\cosh\pi\lambda}{\quad}\right\}\right]$$

$$=\frac{\left(\frac{f\pi^2}{16}\right)\left\{1-\frac{\lambda^4}{(1+\lambda^2)^2}\right\}\frac{\lambda^2}{\pi(1+4\lambda^2)^2}}{\left(1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2}\left[-4\left(\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)(\cosh 2\pi\lambda-1)-4\sinh 2\pi\lambda\sinh\pi\lambda\cosh\pi\lambda\right.$$

$$+\cosh 2\pi\lambda(\cosh 2\pi\lambda-1)+6\left(\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2+3\left(\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)(\cosh 2\pi\lambda+1)$$

$$+3\left(\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)(\cosh 2\pi\lambda-1)$$

$$-2\frac{\sinh 2\pi\lambda}{2\pi\lambda}-2(2\pi^2\lambda^2+5)\left(\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2$$

$$+2\cosh 2\pi\lambda\frac{\sinh 2\pi\lambda}{2\pi\lambda}+2\cosh 2\pi\lambda\cosh\pi\lambda\cosh\pi\lambda$$

$$\left.-2\frac{\sinh 4\pi\lambda}{4\pi\lambda}\right]$$

$$= \frac{\left(\frac{f\pi^2}{16}\right)\left\{1-\frac{\lambda^4}{(1+\lambda^2)^2}\right\}\frac{\lambda^2}{\pi(1+4\lambda^2)^2}}{\left(1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2}\Bigg[-4\left(\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)(\cosh 2\pi\lambda - 1)-(\sinh 2\pi\lambda)^2+(\cosh 2\pi\lambda+1)$$

$$+6\left(\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)\cosh 2\pi\lambda+(\cosh 2\pi\lambda)^2-\cosh 2\pi\lambda$$

$$+6\left(\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2+2\cosh 2\pi\lambda\left(\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)+2\frac{\sinh 2\pi\lambda}{2\pi\lambda}-(4\pi^2\lambda^2+4)\left(\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2-2\frac{\sinh 2\pi\lambda}{2\pi\lambda}\cosh 2\pi\lambda\Bigg]$$

$$= \frac{\left(\frac{f\pi^2}{4}\right)\left\{1-\frac{\lambda^4}{(1+\lambda^2)^2}\right\}\frac{\lambda^2}{\pi(1+4\lambda^2)^2}}{\left(1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2}\left[1+\left\{1+\cosh 2\pi\lambda-(2\pi^2\lambda^2+2)\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right\}\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right]$$

$$\frac{\left\{\frac{4\lambda^4}{\lambda(1+4\lambda^2)^2}\right\}^2}{\left(1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2}\Bigg[2\left(\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)(\cosh 2\pi\lambda-1)-2\left\{3\left(\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)+\cosh 2\pi\lambda\right\}\frac{\sinh 2\pi\lambda}{2\pi\lambda}$$

$$+\left\{1+(2\pi^2\lambda^2+5)\frac{\sinh 2\pi\lambda}{2\pi\lambda}+\cosh 2\pi\lambda\right\}\frac{1}{2\pi^2\lambda^2}(\cosh 2\pi\lambda+1)\Bigg]$$

$$= \frac{\left\{\frac{4\lambda^4}{\lambda(1+4\lambda^2)^2}\right\}^2}{\left(1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2}\Bigg[-2\left(1+3\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)\frac{\sinh 2\pi\lambda}{2\pi\lambda}+\frac{\sinh 2\pi\lambda}{2\pi\lambda}(\cosh 2\pi\lambda+1)$$

$$\left\{1+5\frac{\sinh 2\pi\lambda}{2\pi\lambda}+\cosh 2\pi\lambda\right\}\frac{(\cosh 2\pi\lambda+1)}{2\pi^2\lambda^2}$$

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$$= \frac{\left\{\frac{4\lambda^2}{(1+4\lambda^2)^2}\right\}^2}{\left(1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2} \left[\frac{\sinh 2\pi\lambda}{2\pi\lambda}\lambda^2\left(\cosh 2\pi\lambda - 1 - 6\,\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right) + \frac{1}{2\pi^2}\left(1 + 5\,\frac{\sinh 2\pi\lambda}{2\pi\lambda} + \cosh 2\pi\lambda\right) \cdot (\cosh 2\pi\lambda + 1) \right]$$

Terms depends linearly on a_1, b_1,

$$\lambda^4 \frac{f\pi^2}{64}\left[- \left\{(a_1 - b_1)\frac{\sinh\pi\lambda}{\pi\lambda} + b_1\cosh\pi\lambda\right\} + \frac{\lambda^3}{(1+\lambda^2)^2}\left\{\frac{\sinh\lambda\pi}{\pi}a_1 + \lambda\cosh\pi\lambda\, b_1 - \frac{\sinh\lambda\pi}{\pi}b_1\right\}\right.$$

$$\left. - \frac{4\lambda^6}{(1+4\lambda^2)^2}\left[\frac{\cosh\pi\lambda}{\pi}(a_1 + 2b_1) + \lambda\sinh\pi\lambda\, b_1\right]\right]$$

$$\frac{\frac{(f\pi^2)^2}{32\times 64}\left\{1 - \frac{\lambda^4}{(1+\lambda^2)^2}\right\}}{1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}}\left[-2\left(\frac{\sinh\pi\lambda}{\pi\lambda}\right)^2 + \frac{\lambda^4}{(1+\lambda^2)^2}\left\{\frac{\sinh\lambda\pi}{\pi\lambda}\left(\frac{\sinh\pi\lambda}{\pi\lambda} + \cosh\pi\lambda\right) - \cosh\pi\lambda\cdot\frac{\sinh\pi\lambda}{\pi\lambda} + \frac{\sinh\lambda\pi}{\pi\lambda}\cdot\frac{\sinh\pi\lambda}{\pi\lambda}\right\}\right.$$

$$= \frac{\frac{(f\pi^2)^2}{2\times(32)^2}\left\{1 - \frac{\lambda^4}{(1+\lambda^2)^2}\right\}}{1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}}\left[-2\left(\frac{\sinh\pi\lambda}{\pi\lambda}\right)^2 + \frac{\lambda^4}{(1+\lambda^2)^2}\,2\left(\frac{\sinh\pi\lambda}{\pi\lambda}\right)^2\right]$$

$$= -\frac{(f\pi^2)^2}{(32)^2}\left\{1 - \frac{\lambda^4}{(1+\lambda^2)^2}\right\}^2 \frac{\left(\frac{\sinh\pi\lambda}{\pi\lambda}\right)^2}{1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}}$$

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$$\frac{1}{\left(1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)}(f\pi^2)\left[\frac{\lambda^4}{8\lambda(1+4\lambda^2)^2}\left\{-\left[\left(-\sinh\pi\lambda-\frac{\cosh\pi\lambda}{\pi\lambda}\right)\frac{\sinh\pi\lambda}{\pi\lambda}+\frac{(\cosh\pi\lambda)^2}{\pi\lambda}\right]\right.\right.$$

$$\left.+\frac{\lambda^3}{(1+\lambda^2)^2}\left[-\frac{(\sinh\lambda\pi)^2}{\pi}+\frac{1}{\pi}(\cosh\pi\lambda)^2-\frac{1}{\pi}\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right]\right\}$$

$$-\frac{1}{8}\frac{\lambda^2}{(1+4\lambda^2)^2}\left\{1-\frac{\lambda^4}{(1+\lambda^2)^2}\right\}\left\{\frac{\cosh\pi\lambda}{\pi}\left(\cosh\pi\lambda-\frac{\sinh\pi\lambda}{\pi\lambda}\right)-\frac{1}{\pi}(\sinh\pi\lambda)^2\right\}$$

$$=\frac{(-f\pi^2)}{\left(1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)}\left[\frac{\lambda^2}{8\pi(1+4\lambda^2)^2}\left\{\left(-1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)+\frac{\lambda^4}{(1+\lambda^2)^2}\left(1-\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)\right\}\right.$$

$$\left.-\frac{1}{8\pi}\frac{\lambda^2}{(1+4\lambda^2)^2}\left\{1-\frac{\lambda^4}{(1+\lambda^2)^2}\right\}\left\{1-\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right\}\right]$$

$$=-\frac{(f\pi^2)}{\left(1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)}\frac{\lambda^2}{4\pi(1+4\lambda^2)^2}\left\{1-\frac{\lambda^4}{(1+\lambda^2)^2}\right\}\left\{1-\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right\}$$

$$- \frac{\frac{32\lambda^6}{\lambda(1+4\lambda^2)^4}}{1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}} \left[\frac{\cosh\pi\lambda}{\pi} \left\{ -\sinh\pi\lambda + 2\frac{\cosh\pi\lambda}{\pi\lambda} \right\} + \frac{\sinh\pi\lambda \cancel{\cosh\pi\lambda}}{\pi} \right]$$

$$= - \frac{32\lambda^4}{\pi^2(1+4\lambda^2)^2} \frac{(\cosh 2\pi\lambda + 1)}{1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}}$$

$$H_1(\lambda) = \frac{17}{4(128)^2} + \frac{\lambda^4}{4(32)^2} + \frac{\lambda^4}{8(32)^2(1+\lambda^2)^2} - \cancel{\frac{1}{16(32)^2} \frac{\left(\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2}{1+\frac{\sinh 4\pi\lambda}{4\pi\lambda}}}$$

$$+ \frac{1}{64(32)^2} \frac{\frac{\sinh 2\pi\lambda}{2\pi\lambda}}{\left(1+\frac{\sinh 4\pi\lambda}{4\pi\lambda}\right)^2} \left\{ (8\pi^2\lambda^2 \overset{-1}{\pm 3}) \frac{\sinh 2\pi\lambda}{2\pi\lambda} \frac{\sinh 4\pi\lambda}{4\pi\lambda} + 2\left(\cosh 2\pi\lambda \overset{-}{\oplus} \frac{\sinh 2\pi\lambda}{2\pi\lambda}\right) \right\}$$

$$+ \frac{1}{4(32)^2} \left\{ 1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right\}^2 \frac{\frac{\sinh\pi\lambda}{\pi\lambda}}{\left(1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2} \left\{ (9\pi^2\lambda^2 \overset{-1}{\pm 3}) \frac{\sinh\pi\lambda}{\pi\lambda} \frac{\sinh 2\pi\lambda}{2\pi\lambda} + 2\left(\cosh\pi\lambda \overset{-}{\oplus} \frac{\sinh\pi\lambda}{\pi\lambda}\right) \right\}$$

$$\cancel{- \frac{1}{(32)^2} \left\{ 1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right\}^2 \frac{\left(\frac{\sinh\pi\lambda}{\pi\lambda}\right)^2}{1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}}}$$

$$H_2(\lambda) = \frac{1}{8\pi}\left\{1-\frac{\lambda^4}{(1+\lambda^2)^2}\right\}\frac{\lambda^2}{(1+4\lambda^2)^2}\,\frac{\left[-1+\left\{1+\cosh 2\pi\lambda-(2\pi^2\lambda^2+1)\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right\}\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right]}{\left(1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2}$$

$$\cancel{-\frac{1}{4\pi}\left\{1-\frac{\lambda^4}{(1+\lambda^2)^2}\right\}\frac{\lambda^2}{(1+4\lambda^2)}\,\frac{1-\frac{\sinh 2\pi\lambda}{2\pi\lambda}}{1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}}}$$

$$H_2(\lambda) = \frac{1}{8\pi}\left\{1-\frac{\lambda^4}{(1+\lambda^2)^2}\right\}\frac{\lambda^2}{(1+4\lambda^2)^2}\,\frac{\frac{\sinh 2\pi\lambda}{2\pi\lambda}\left\{1+\cosh 2\pi\lambda-2\pi^2\lambda^2\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right\}-1}{\left(1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2}$$

$$H_3(\lambda) = \frac{2\lambda^4}{(1+4\lambda^2)^2}+\left(\frac{4\lambda^2}{(1+4\lambda^2)^2}\right)^2\frac{\left[\lambda^2\frac{\sinh 2\pi\lambda}{2\pi\lambda}\left(\cosh 2\pi\lambda-1-6\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)+\frac{1}{2\pi^2}\left(1+5\frac{\sinh 2\pi\lambda}{2\pi\lambda}+\cosh 2\pi\lambda\right)\left(\cosh 2\pi\lambda+1\right)\right]}{\left(1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2}$$

$$-\frac{32}{\pi^2}\frac{\lambda^4}{(1+4\lambda^2)^4}\,\frac{(1+\cosh 2\pi\lambda)}{1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}}$$

$$K = 2\left\{512\, G_1 G_3 \left(\frac{\delta}{t}\right)^2 + G_3\left(\frac{1}{8} + \frac{\lambda^2}{3} + \frac{2\lambda^4}{3}\right)\right\}^{\frac{1}{2}} + 24\, G_2 \left(\frac{\delta}{t}\right) \qquad \underline{\underline{516}}$$

$$\left(\frac{a}{R}\right)^2 = \left(\frac{t}{R}\right)\pi^2 \left\{\frac{8G_1}{G_3}\left(\frac{\delta}{t}\right)^2 + \frac{1}{G_3}\left(\frac{1}{512} + \frac{\lambda^2}{192} + \frac{\lambda^4}{96}\right)\right\}^{\frac{1}{2}} \underline{\underline{\frac{1}{\lambda^2}}} \qquad !!!$$

$$G_1(\lambda) = \frac{17}{65536} + \frac{\lambda^4}{4096} + \frac{1}{8192}\frac{\lambda^4}{(1+\lambda^2)^2}$$

$$+ \frac{1}{65536}\frac{\frac{\sinh 2\pi\lambda}{2\pi\lambda}}{\left(1+\frac{\sinh 4\pi\lambda}{4\pi\lambda}\right)^2}\left\{(8\pi^2\lambda^2-1)\frac{\sinh 2\pi\lambda}{2\pi\lambda}\frac{\sinh 4\pi\lambda}{4\pi\lambda} + 2\left(\cosh 2\pi\lambda - \frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)\right\}$$

$$+ \frac{1}{4096}\left\{1-\frac{\lambda^4}{(1+\lambda^2)^2}\right\}\frac{2\,\frac{\sinh \pi\lambda}{\pi\lambda}}{\left(1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2}\left\{(2\pi^2\lambda^2-1)\frac{\sinh \pi\lambda}{\pi\lambda}\frac{\sinh 2\pi\lambda}{2\pi\lambda} + 2\left(\cosh \pi\lambda - \frac{\sinh \pi\lambda}{\pi\lambda}\right)\right\}$$

$$G_2(\lambda) = \frac{1}{8\pi}\left\{1-\frac{\lambda^4}{(1+\lambda^2)^2}\right\}\frac{1}{(1+4\lambda^2)^2}\,\frac{\frac{\sinh 2\pi\lambda}{2\pi\lambda}\left\{1+\cosh 2\pi\lambda - 2\pi^2\lambda^2\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right\}-1}{\left(1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2}$$

$$G_3(\lambda) = \frac{2}{(1+4\lambda^2)^2}$$

$$+\left\{\frac{4}{(1+4\lambda^2)^2}\right\}^2 \frac{\left[\lambda^2\frac{\sinh 2\pi\lambda}{2\pi\lambda}\left(\cosh 2\pi\lambda - 1 - 6\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right) + \frac{1}{2\pi^2}\left(-3+\frac{\sinh 2\pi\lambda}{2\pi\lambda}+\cosh 2\pi\lambda\right)(\cosh 2\pi\lambda+1)\right]}{\left(1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2}$$

$$\frac{\sinh 2\pi}{2\pi} = 42.613218, \quad \cosh 2\pi = 267.74862$$

$$\frac{\sinh 4\pi}{4\pi} = 11409.473, \quad \cosh 4\pi = 143375.66$$

$$\frac{\sinh 8\pi}{8\pi} = 1635840500, \quad \cosh 8\pi = 41113157000$$

for $\lambda = 1.000$ $\qquad \frac{\sinh \pi\lambda}{\pi\lambda} = 3.676164$

$$G_1(\lambda) = 0.000259399 + 0.000244141 + 0.0\ 030518$$

$$+\ 0.0000152588\ \frac{42.613218}{11410.473^2}\left\{77.956835 \times 4\ \ 13218 \times 11409.473 + 2 \times 225.13540\right\}$$

$$+\ 0.000244141 \times 0.5625 \times \frac{3.676164}{43.613218^2}\left\{18.7392\ \ \times 3.676164 \times 42.613218 + 2 \times 7.91584\right\}$$

$$= 0.000259399 + 0.000244141 + 0.000030518$$

$$+\ 0.0000152588\ \frac{42.613218 \times 37902624}{\underset{1.301989}{1140.473^2}} + 0.0002\ \ 141 \times 0.5625 \times \frac{3.676164 \times 2951.360}{\underset{0.19021128}{43.613218^2}}$$

$$= 0.000259399 + 0.000244141 + 0.000030518 + 0.000\ldots 9290 + 0.000783329$$

$$= \underline{\underline{0.00150168}}$$

$$G_2(\lambda) = 0.03978873 \times 0.75 \times 0.04\ \frac{42.613218\left\{268.74462 - 841.15121\right\} - 1}{43.613218^2}$$

$$= -0.03978873 \times 0.75 \times 0.04\ \frac{24390.916}{\underset{0.19021128}{43.613218}} = \underline{\underline{-0.0153076}}$$

$$G_3(\lambda) = 0.08$$

$$+\ 0.0256\ \frac{42.613218 \times 11.06930 + \frac{1}{2\pi^2} \times 307.36184 \times 268.74462}{43.613218^2}$$

$$= 0.08 + 0.0256\ \frac{4656.4189}{43.613218^2}$$

$$= 0.08 + 0.062669 = \underline{\underline{0.142669}}$$

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$$\boxed{K = 2\left\{0.110058\left(\frac{\delta}{E}\right)^2 + 0.160503\right\}^{\frac{1}{2}} - 0.367382\left(\frac{\delta}{E}\right) \qquad \lambda = 1.00}$$

$K_0 = \underline{\underline{0.80126}}$

$$0.220116\left(\frac{\delta}{E}\right) = 0.367382\left\{0.110058\left(\frac{\delta}{E}\right)^2 + 0.160503\right\}^{\frac{1}{2}}$$

$$0.0484511\left(\frac{\delta}{E}\right)^2 = 0.0216131$$

$$\begin{array}{r} 0.0148545 \\ \hline 0.0335966 \end{array}$$

$$\left(\frac{\delta}{E}\right)^2 = 0.644800$$

$$\left(\frac{\delta}{E}\right) = 0.80300$$

$K_{min} = \underline{\underline{0.66722}}$

$\lambda = 0.5$

$$\frac{\sinh 0.5\pi}{0.5\pi} = 1.46505, \qquad \cosh 0.5\pi = 2.50920$$

$$\frac{\sinh \pi}{\pi} = 3.676164, \qquad \cosh \pi = 11.5920$$

$$G_1(\lambda) = 0.000259399 + 0.000015259 + 0.000004883$$

$$+ 0.0000152588 \times \frac{3.676164}{43.613218^2}\left\{18.739209 \times 3.67614 \times 42.613218 + 2 \times 29.1564\right\}$$

$$+ 0.000244141 \times 0.9216 \times \frac{1.46505}{4.676164^2}\left\{3.9348022 \times 1.46505 \times 3.676164 + 2 \times 1.04415\right\}$$

$$= 0.000259399 + 0.000015259 + 0.000004883 + 0.0000152588 \times \frac{10849.7171}{43.613218^2}$$

$$+ 0.000244141 \times 0.9216 \times \frac{34.106680}{\underset{21.866510}{4.676164^2}}$$

$$= 0.000259399 + 0.000015259 + 0.000004883 + 0.000087037 + 0.000350948$$

$$= \underline{\underline{0.000717526}}$$

$$G_2(1) = 0.03978873 \times 0.96 \times 0.25 \times \frac{3.676164\{12.5920 - 18.141142\} - 1}{4.676164^2}$$

$$= -0.03978873 \times 0.96 \times 0.25 \times \frac{21.39956}{4.676164^2} = \underline{\underline{-0.00934538}}$$

$$G_3(1) = 0.5 + \frac{-0.25 \times 3.676164 \times 11.46984 + 7.826085}{4.676164^2}$$

$$= 0.5 - \frac{2.715168}{4.676164^2} = \underline{\underline{0.375830}}$$

$$\boxed{K = 2\left\{0.138070\left(\frac{f}{\ell}\right)^2 + 0.0939575\right\}^{\frac{1}{2}} - 0.224289\left(\frac{f}{\ell}\right) \qquad \lambda = 0.5}$$

$K_0 = \underline{\underline{0.61305}}$

$$0.276140\left(\frac{f}{\ell}\right) = 0.224289\left\{0.138070\left(\frac{f}{\ell}\right)^2 + 0.0939575\right\}^{\frac{1}{2}}$$

$$0.0762533\left(\frac{f}{\ell}\right)^2 = 0.0047266$$

$$\begin{array}{r} 0.0069457 \\ \hline 0.0693076 \end{array}$$

$$\left(\frac{f}{\ell}\right)^2 = 0.0681921$$

$$\left(\frac{f}{\ell}\right) = 0.261146$$

$K_{min.} = \underline{\underline{0.58446}}$

$$\lambda = 1.5$$

$$\log_{10}\left(e^{1.5\pi}\right) = 0.434294482 \times 1.5\pi = 2.0465646$$

$$e^{1.5\pi} = 111.31779, \qquad e^{-1.5\pi} = 0.00898$$

$$\sinh 1.5\pi = 55.654405 \qquad \cosh 1.5\pi = 55.663385$$

$$\frac{\sinh 1.5\pi}{1.5\pi} = 11.810231$$

$$\log_{10}(e^{3.0\pi}) = 0.434294482 \times 9.4247781 = 4.0931291$$

$$e^{3\pi} = 12391.650\,, \qquad e^{-3\pi} = 0.000$$

$$\sinh 3.0\pi \cong \cosh 3.0\pi = 6195.8250$$

$$\frac{\sinh 3\pi}{3\pi} = 657.39745$$

$$\sinh 6\pi \cong \cosh 6\pi = 76776495$$

$$\frac{\sinh 6\pi}{6\pi} = 4073119.5$$

$$\frac{\sinh 1.5\pi}{1.5\pi} = 11.810231 \qquad \cosh 1.5\pi = 55.663385$$

$$\frac{\sinh 3\pi}{3\pi} = 657.39745 \qquad \cosh 3\pi = 6195.8250$$

$$\frac{\sinh 6\pi}{6\pi} = 4073119.5 \qquad \cosh 6\pi = 76776495$$

$$G_1(\lambda) = 0.000259399 + 0.001235962 + 0.000058507$$

$$+ 0.0000152588 \times \frac{657.39745}{4073120.5^2}\left\{176.65288 \times 657.39745 \times 4073119.5 + 11076.455\right\}$$

$$+ 0.000244141 \times 0.27113897 \times \frac{11.810231}{658.39745^2}\left\{43.413220 \times 11.810231 \times 657.39745 + 87.706308\right\}$$

$$= 0.000259399 + 0.001235962 + 0.000058507$$

$$+ 0.0000152588 \times \frac{31095.956}{40.731205^2} + 0.000244141 \times 0.27113897 \times \frac{398.18032}{658.39745^2}$$

1659.03106 43.3…

$$= 0.000259399 + 0.001235962 + 0.000058507 + 0.000285954 + 0.000608041 = \underline{0.002447861}$$

$$G_2(\lambda) = -0.03978873 \times 0.52071006 \times 0.01 \, \frac{657.39745 \times 23000.313 \overset{0.0001}{+1}}{658.39745^2} \quad \underline{\underline{531}}$$

$$= -0.03978873 \times 0.52071006 \times 0.01 \times 34.880726 = \underline{\underline{-0.00722673}}$$

$$G_3(\lambda) = 0.02 + 0.0016 \, \frac{2.25 \times 657.39745 \times 2250.4400 + 2181916.9}{658.39745^2}$$

$$= 0.02 + 0.0016 \, \frac{551.06423}{43.348720}$$

$$= 0.02 + 0.0203398 = \underline{\underline{0.0403398}}$$

$$\frac{1}{8} + \frac{\lambda^2}{3} + \frac{2\lambda^4}{3} = \frac{1}{8} + \frac{1}{3}\lambda^2(1+2\lambda^2) = 0.125 + 4.125 = \underline{4.2500}$$

$$K = 2\left\{0.0505582\left(\frac{\delta}{E}\right)^2 + 0.171444\right\}^{\frac{1}{2}} - 0.173442\left(\frac{\delta}{E}\right)$$

$$K_0 = \underline{\underline{0.82812}}$$

$$0.1011164\left(\frac{\delta}{E}\right) = 0.173442\left\{0.0505582\left(\frac{\delta}{E}\right)^2 + 0.171444\right\}$$

$$\begin{array}{r} 0.01022453 \\ 0.00152090 \\ \hline 0.00870363 \end{array} \left(\frac{\delta}{E}\right)^2 = 0.005157408$$

$$\left(\frac{\delta}{E}\right)^2 = 0.592558$$

$$\left(\frac{\delta}{E}\right) = 0.76978$$

$$K_{min} = \underline{\underline{0.76405}}$$

$$\frac{w}{R}=\frac{1}{2}\left(\frac{a}{R}\right)^2\left[1-\left(\frac{x}{a}\right)^2-\frac{f}{4}\left(1+\cos\frac{\pi x}{a}\right)\left(1+\cos\frac{\pi y}{b}\right)\right]$$

$$\frac{w_0}{R}=\frac{1}{2}\left(\frac{a}{R}\right)^2\left[1-\left(\frac{x}{a}\right)^2\right]$$

$$\frac{\partial w}{\partial x}=\left(\frac{a}{R}\right)\left\{-\left(\frac{x}{a}\right)+\frac{f}{8}\pi\sin\frac{\pi x}{a}\left(1+\cos\frac{\pi y}{b}\right)\right\}$$

$$\frac{\partial w}{\partial y}=\left(\frac{a}{R}\right)\left\{\qquad+\frac{f}{8}\pi\lambda\left(1+\cos\frac{\pi x}{a}\right)\sin\frac{\pi y}{b}\right\}$$

$$\frac{\partial^2 w}{\partial x^2}=\frac{1}{R}\left\{-1+\frac{f}{8}\pi^2\cos\frac{\pi x}{a}\left(1+\cos\frac{\pi y}{b}\right)\right\}$$

$$\frac{\partial^2 w}{\partial y^2}=\frac{1}{R}\left\{\qquad+\frac{f}{8}\pi^2\lambda^2\left(1+\cos\frac{\pi x}{a}\right)\cos\frac{\pi y}{b}\right\}$$

$$\frac{\partial^2 w}{\partial x\partial y}=\frac{1}{R}\left\{\qquad-\frac{f}{8}\pi^2\lambda\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}\right\}$$

$$\nabla\nabla F=\frac{1}{R^2}E\frac{f\pi^2\lambda^2}{8}\left\{\frac{f\pi^2}{32}\left[\left(1-\cos\frac{2\pi x}{a}\right)\left(1-\cos\frac{2\pi y}{b}\right)-\left(1+2\cos\frac{\pi x}{a}+\cos\frac{2\pi x}{a}\right)\left(1+2\cos\frac{\pi y}{b}+\cos\frac{2\pi y}{b}\right)\right]+\cos\frac{\pi y}{b}+\cos\frac{\pi x}{a}\cos\frac{\pi y}{b}\right\}$$

$$=\frac{E}{R^2}\frac{f\pi^2\lambda^2}{8}\left\{\frac{f\pi^2}{32}\left[-2\cos\frac{2\pi x}{a}-2\cos\frac{2\pi y}{b}-2\cos\frac{\pi x}{a}-2\cos\frac{\pi y}{b}-4\cos\frac{\pi x}{a}\cos\frac{\pi y}{b}-2\cos\frac{2\pi x}{a}\cos\frac{\pi y}{b}-2\cos\frac{\pi x}{a}\cos\frac{2\pi y}{b}\right]+\cos\frac{\pi y}{b}+\cos\frac{\pi x}{a}\cos\frac{\pi y}{b}\right\}$$

$$=\frac{E}{R^2}\frac{f\pi^2\lambda^2}{8}\left[\left(1-\frac{f\pi^2}{16}\right)\cos\frac{\pi y}{b}+\left(1-\frac{f\pi^2}{8}\right)\cos\frac{\pi x}{a}\cos\frac{\pi y}{b}-\frac{f\pi^2}{16}\left\{\cos\frac{2\pi x}{a}+\cos\frac{2\pi y}{b}+\cos\frac{\pi x}{a}+\cos\frac{\pi x}{a}\cos\frac{2\pi y}{b}+\cos\frac{2\pi x}{a}\cos\frac{\pi y}{b}\right\}\right]$$

The particular integral is

$$F^{(1)} = \frac{E}{R^2}\frac{f\pi^2\lambda^2}{8}\Bigg[\left(1-\frac{f\pi^2}{16}\right)\frac{\cos\frac{\pi y}{b}}{\left(\frac{\pi}{b}\right)^4} + \left(1-\frac{f\pi^2}{8}\right)\frac{\cos\frac{\pi x}{a}\cos\frac{\pi y}{b}}{\left(\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2\right)^2}$$

$$-\frac{f\pi^2}{16}\left\{\frac{\cos\frac{2\pi x}{a}}{\left(\frac{2\pi}{a}\right)^4} + \frac{\cos\frac{2\pi y}{b}}{\left(\frac{2\pi}{b}\right)^4} + \frac{\cos\frac{\pi x}{a}}{\left(\frac{\pi}{a}\right)^4} + \frac{\cos\frac{\pi x}{a}\cos\frac{2\pi y}{b}}{\left(\left(\frac{\pi}{a}\right)^2+\left(\frac{2\pi}{b}\right)^2\right)^2} + \frac{\cos\frac{2\pi x}{a}\cos\frac{\pi y}{b}}{\left(\left(\frac{2\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2\right)^2}\right\}\Bigg]$$

$$F = E\left(\frac{a}{R}\right)^2\frac{f\lambda^2}{8}\frac{1}{\left(\frac{\pi}{a}\right)^2}\Bigg[\left(1-\frac{f\pi^2}{16}\right)\frac{1}{\lambda^4}\cos\frac{\pi y}{b} + \left(1-\frac{f\pi^2}{8}\right)\frac{\cos\frac{\pi x}{a}\cos\frac{\pi y}{b}}{(1+\lambda^2)^2}$$

$$-\frac{f\pi^2}{16}\left\{\cos\frac{\pi x}{a} + \frac{1}{16}\cos\frac{2\pi x}{a} + \frac{1}{16\lambda^4}\cos\frac{2\pi y}{b} + \frac{\cos\frac{\pi x}{a}\cos\frac{2\pi y}{b}}{(1+4\lambda^2)^2} + \frac{\cos\frac{2\pi x}{a}\cos\frac{\pi y}{b}}{(4+\lambda^2)^2}\right\}$$

$$+ a_0\left(\frac{\pi x}{a}\right)^2 + \left\{a_1\cosh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\right\}\cos\frac{\pi y}{b}$$

$$+\left\{a_2\cosh\frac{2\pi\lambda x}{a} + b_2\left(\frac{2\pi\lambda x}{a}\right)\sinh\frac{2\pi\lambda x}{a}\right\}\cos\frac{2\pi y}{b}\Bigg]$$

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} = E\left(\frac{a}{R}\right)^2 \frac{f\lambda^2}{8}\Bigg[\left(\frac{f\pi^2}{16}-1\right)\frac{1}{\lambda^2}\cos\frac{\pi y}{b} + \left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^2}{(1+\lambda^2)^2}\cos\frac{\pi x}{a}\cos\frac{\pi y}{b}$$

$$+\frac{f\pi^2}{16}\left\{\frac{1}{4\lambda^2}\cos\frac{2\pi y}{b} + \frac{4\lambda^2}{(1+4\lambda^2)^2}\cos\frac{\pi x}{a}\cos\frac{2\pi y}{b} + \frac{\lambda^2}{(4+\lambda^2)^2}\cos\frac{2\pi x}{a}\cos\frac{\pi y}{b}\right\}$$

$$-\lambda^2\left\{a_1\cosh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\right\}\cos\frac{\pi y}{b}$$

$$-4\lambda^2\left\{a_2\cosh\frac{2\pi\lambda x}{a} + b_2\left(\frac{2\pi\lambda x}{a}\right)\sinh\frac{2\pi\lambda x}{a}\right\}\cos\frac{2\pi y}{b}\Bigg]$$

$$= E\left(\frac{a}{R}\right)^2\frac{f\lambda^2}{8}\Bigg\{\Bigg[\left(\frac{f\pi^2}{16}-1\right)\frac{1}{\lambda^2} + \left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^2}{(1+\lambda^2)^2}\cos\frac{\pi x}{a} + \frac{f\pi}{16}\frac{\lambda^2}{(4+\lambda^2)^2}\cos\frac{2\pi x}{a}$$

$$-\lambda^2\left\{a_1\cosh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\right\}\Bigg]\cos\frac{\pi y}{b}$$

$$+\left[\frac{f\pi^2}{16}\left\{\frac{1}{4\lambda^2} + \frac{4\lambda^2}{(1+4\lambda^2)^2}\cos\frac{\pi x}{a}\right\} - 4\lambda^2\left\{a_2\cosh\frac{2\pi\lambda x}{a} + b_2\left(\frac{2\pi\lambda x}{a}\right)\sinh\frac{2\pi\lambda x}{a}\right\}\right]\cos\frac{2\pi y}{b}\Bigg\}$$

$$a_1\cosh\pi\lambda + b_1(\pi\lambda)\sinh\pi\lambda = \frac{1}{\lambda^4}\left[\left(\frac{f\pi^2}{16}-1\right) - \left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^4}{(1+\lambda^2)^2} + \frac{f\pi^2}{16}\frac{\lambda^4}{(4+\lambda^2)^2}\right]$$

$$a_2\cosh 2\pi\lambda + b_2(2\pi\lambda)\sinh 2\pi\lambda = \frac{1}{16\lambda^4}\left[\frac{f\pi^2}{16}\left\{1 - \frac{16\lambda^4}{(1+4\lambda^2)^2}\right\}\right]$$

Conditions for free edges!

$$\tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y} = E\left(\frac{a}{R}\right)^2 \frac{f\lambda^2}{8}\Bigg[\left(\frac{f\pi^2}{8}-1\right)\frac{\lambda}{(1+\lambda^2)^2}\sin\frac{\pi x}{a}\sin\frac{\pi y}{b} + \frac{f\pi^2}{16}\left\{\frac{2\lambda}{(1+4\lambda^2)^2}\sin\frac{\pi x}{a}\sin\frac{2\pi y}{b} + \frac{2\lambda}{(4+\lambda^2)^2}\sin\frac{2\pi x}{a}\sin\frac{\pi y}{b}\right\}$$

$$+\lambda^2\left\{(a_1+b_1)\sinh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\cosh\frac{\pi\lambda x}{a}\right\}\sin\frac{\pi y}{b}$$

$$+4\lambda^2\left\{(a_2+b_2)\sinh\frac{2\pi\lambda x}{a} + b_2\left(\frac{2\pi\lambda x}{a}\right)\cosh\frac{2\pi\lambda x}{a}\right\}\sin\frac{2\pi y}{b}\Bigg]$$

$$= E\left(\frac{a}{R}\right)^2 \frac{f\lambda^2}{8}\Bigg\{\Bigg[\left(\frac{f\pi^2}{8}-1\right)\frac{\lambda}{(1+\lambda^2)^2}\sin\frac{\pi x}{a} + \frac{f\pi^2}{16}\frac{2\lambda}{(4+\lambda^2)^2}\sin\frac{2\pi x}{a}$$

$$+\lambda^2\left\{(a_1+b_1)\sinh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\cosh\frac{\pi\lambda x}{a}\right\}\Bigg]\sin\frac{\pi y}{b}$$

$$+\Bigg[\frac{f\pi^2}{16}\frac{2\lambda}{(1+4\lambda^2)^2}\sin\frac{\pi x}{a} + 4\lambda^2\left\{(a_2+b_2)\sinh\frac{2\pi\lambda x}{a} + b_2\left(\frac{2\pi\lambda x}{a}\right)\cosh\frac{2\pi\lambda x}{a}\right\}\Bigg]\sin\frac{2\pi y}{b}\Bigg\}$$

$$\boxed{\begin{aligned}&(a_1+b_1)\sinh\pi\lambda + b_1(\pi\lambda)\cosh\pi\lambda = 0\\&(a_2+b_2)\sinh 2\pi\lambda + b_2(2\pi\lambda)\cosh 2\pi\lambda = 0\end{aligned}}$$

$$\sigma_y = \frac{\partial^2 F}{\partial x^2} = E\left(\frac{a}{R}\right)^2 \frac{f\lambda^2}{8}\Bigg[\left(\frac{f\pi^2}{8}-1\right)\frac{1}{(1+\lambda^2)^2}\cos\frac{\pi x}{a}\cos\frac{\pi y}{b} + \frac{f\pi^2}{16}\Big\{\cos\frac{\pi x}{a} + \frac{1}{4}\cos\frac{2\pi x}{a} + \frac{\cos\frac{\pi x}{a}\cos\frac{2\pi y}{b}}{(1+4\lambda^2)^2} +$$

$$+ \frac{4}{(4+\lambda^2)^2}\cos\frac{2\pi x}{a}\cos\frac{\pi y}{b}\Big\} + 2a_0 + \lambda^2\left\{(a_1+2b_1)\cosh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\right\}\cos\frac{\pi y}{b} +$$

$$+ 4\lambda^2\left\{(a_2+2b_2)\cosh\frac{2\pi\lambda x}{a} + b_2\left(\frac{2\pi\lambda x}{a}\right)\sinh\frac{2\pi\lambda x}{a}\right\}\cos\frac{2\pi y}{b}\Bigg]$$

$$= E\left(\frac{a}{R}\right)^2 \frac{f\lambda^2}{8}\Bigg[\frac{f\pi^2}{16}\cos\frac{\pi x}{a} + \frac{f\pi^2}{64}\cos\frac{2\pi x}{a} + 2a_0$$

$$+ \left\{\left(\frac{f\pi^2}{8}-1\right)\frac{\cos\frac{\pi x}{a}}{(1+\lambda^2)^2} + \frac{f\pi^2}{16}\frac{4}{(4+\lambda^2)^2}\cos\frac{2\pi x}{a} + \lambda^2\left[(a_1+2b_1)\cosh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\right]\right\}\cos\frac{\pi y}{b}$$

$$+ \left\{\frac{f\pi^2}{16}\frac{1}{(1+4\lambda^2)^2}\cos\frac{\pi x}{a} + 4\lambda^2\left[(a_2+2b_2)\cosh\frac{2\pi\lambda x}{a} + b_2\left(\frac{2\pi\lambda x}{a}\right)\sinh\frac{2\pi\lambda x}{a}\right]\right\}\cos\frac{2\pi y}{b}\Bigg]$$

$$\sigma_{y\,y=b} = E\left(\frac{a}{R}\right)^2 \frac{f\lambda^2}{8}\Bigg[\frac{f\pi^2}{16}\cos\frac{\pi x}{a} + \frac{f\pi^2}{64}\cos\frac{2\pi x}{a} + 2a_0$$

$$- \left\{\left(\frac{f\pi^2}{8}-1\right)\frac{\cos\frac{\pi x}{a}}{(1+\lambda^2)^2} + \frac{f\pi^2}{16}\frac{4}{(4+\lambda^2)^2}\cos\frac{2\pi x}{a} + \lambda^2\left[(a_1+2b_1)\cosh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\right]\right\}$$

$$+ \left\{\frac{f\pi^2}{16}\frac{1}{(1+4\lambda^2)^2}\cos\frac{\pi x}{a} + 4\lambda^2\left[(a_2+2b_2)\cosh\frac{\pi\lambda x}{a} + b_2\left(\frac{2\pi\lambda x}{a}\right)\sinh\frac{2\pi\lambda x}{a}\right]\right\}\Bigg]$$

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$$-\frac{\sigma}{E} = \left(\frac{a}{R}\right)^2 \frac{f\lambda^2}{8}\Big[2a_0 - \frac{\lambda}{\pi}\Big\{ (a_1+b_1)\sinh \lambda\pi + b_1 (\pi\lambda)\cosh \pi\lambda \Big\}$$

$$+ \frac{2\lambda}{\pi}\Big\{ (a_2+b_2)\sinh 2\lambda\pi + b_2 (2\pi\lambda)\cosh 2\pi\lambda \Big\}\Big] = \underline{\underline{\left(\frac{a}{R}\right)^2 \frac{f\lambda^2}{8}\big[2a_0\big] = -\frac{\sigma}{E}}}$$

$$\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 = \left(\frac{a}{R}\right)^2 \frac{f\lambda^2}{8}\Big[\frac{f\pi^2}{64}\Big(3+4\cos\frac{\pi x}{a} + \cos\frac{2\pi x}{a}\Big)\Big(1-\cos\frac{2\pi y}{b}\Big)\Big]$$

$$= \left(\frac{a}{R}\right)^2 \frac{f\lambda^2}{8}\Big[\frac{f\pi^2}{64}\Big(3+4\cos\frac{\pi x}{a} + \cos\frac{2\pi x}{a}\Big)$$

$$- \frac{f\pi^2}{64}\Big(3+4\cos\frac{\pi x}{a} + \cos\frac{2\pi x}{a}\Big)\cos\frac{2\pi y}{b}\Big]$$

$$\frac{\partial v}{\partial y} = \left(\frac{a}{R}\right)^2 \frac{f\lambda^2}{8}\Big[-\frac{3f\pi^2}{64} + 2a_0$$

$$+ \Big\{ \Big(\frac{f\pi^2}{8}-1\Big)\frac{\cos\frac{\pi x}{a}}{(1+\lambda^2)^2} + \frac{f\pi^2}{16}\frac{4}{(4+\lambda^2)^2}\cos\frac{2\pi x}{a} + \cdots\cdots \Big\}\cos\frac{\pi y}{b}$$

$$+ \Big\{ \cdots\cdots \Big\}\cos\frac{2\pi y}{b}\Big]$$

$$\left.\frac{v}{b}\right|_{y=b} = \left(\frac{a}{R}\right)^2 \frac{f\lambda^2}{8}\Big[-\frac{3}{64}(f\pi^2) + 2a_0\Big] = -\left(\frac{a}{R}\right)^2 \frac{f\lambda^2}{8}\frac{3}{64}(f\pi^2) - \frac{\sigma}{E}$$

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$$\boxed{\frac{\Delta p}{Eabt} = -\left(\frac{\sigma}{E}\right)^2 - \left(\frac{a}{R}\right)^2\left(\frac{f}{g}\right)^2\left(\frac{3}{8}\pi^2\lambda^2\right)\frac{\sigma}{E}}$$

$$\frac{1}{gab}\int_0^a\int_0^b \left(\frac{\sigma_v}{E}\right)^2 dx\,dy = \left(\frac{a}{R}\right)^4\left(\frac{f}{g}\right)^2\Bigg[\frac{1}{4}\left(\frac{f\pi^2}{16}-1\right)^2 + \frac{1}{8}\left\{\left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^4}{(1+\lambda^2)^2}\right\}^2 + \frac{1}{8}\left\{\frac{f\pi^2}{16}\frac{\lambda^4}{(4+\lambda^2)^2}\right\}^2$$

$$-\frac{1}{2}\lambda^4\left(\frac{f\pi^2}{16}-1\right)\int_0^1\left\{a_1\cosh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\right\}d\left(\frac{x}{a}\right)$$

$$-\frac{1}{2}\lambda^4\left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^4}{(1+\lambda^2)^2}\int_0^1\left\{a_1\cosh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\right\}\cos\frac{\pi x}{a}\,d\left(\frac{x}{a}\right)$$

$$-\frac{1}{2}\lambda^4\,\frac{f\pi^2}{16}\frac{\lambda^4}{(4+\lambda^2)^2}\int_0^1\left\{a_1\cosh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\right\}\cos\frac{2\pi x}{a}\,d\left(\frac{x}{a}\right)$$

$$+\frac{1}{4}\lambda^8\int_0^1\left\{a_1\cosh\frac{\pi\lambda x}{a} + b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\right\}^2 d\left(\frac{x}{a}\right) + \frac{1}{4}\left(\frac{f\pi^2}{64}\right)^2 + \frac{1}{8}\left\{\frac{f\pi^2}{16}\frac{4\lambda^4}{(1+4\lambda^2)^2}\right\}^2$$

$$-2\lambda^4\,\frac{f\pi^2}{64}\int_0^1\left\{a_2\cosh\frac{2\pi\lambda x}{a} + b_2\left(\frac{2\pi\lambda x}{a}\right)\sinh\frac{2\pi\lambda x}{a}\right\}d\left(\frac{x}{a}\right)$$

$$-2\lambda^4\,\frac{f\pi^2}{16}\frac{4\lambda^4}{(1+4\lambda^2)^2}\int_0^1\left\{a_2\cosh\frac{2\pi\lambda x}{a} + b_2\left(\frac{2\pi\lambda x}{a}\right)\sinh\frac{2\pi\lambda x}{a}\right\}\cos\frac{\pi x}{a}\,d\left(\frac{x}{a}\right)$$

$$+\;4\lambda^8\int_0^1\left\{a_2\cosh\frac{2\pi\lambda x}{a} + b_2\left(\frac{2\pi\lambda x}{a}\right)\sinh\frac{2\pi\lambda x}{a}\right\}^2 d\left(\frac{x}{a}\right)\Bigg]$$

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$$\frac{1}{2ab}\int_0^a\int_0^b\left(\frac{\sigma_y}{E}\right)^2 dx\,dy = \left(\frac{a}{R}\right)^4\left(\frac{f}{8}\right)^2\Bigg[\frac{1}{4}\left(\frac{f\pi^2}{16}\lambda^2\right)^2+\frac{1}{4}\left(\frac{f\pi^2}{64}\lambda^2\right)^2+\frac{1}{8}\left\{\left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^2}{(1+\lambda^2)^2}\right\}^2+\frac{1}{8}\left\{\frac{f\pi^2}{16}\frac{4\lambda^2}{(4+\lambda^2)^2}\right\}^2$$

$$+\frac{1}{2}\lambda^4\left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^2}{(1+\lambda^2)^2}\int_0^1\left\{(a_1+2b_1)\cosh\frac{\pi\lambda x}{a}+b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\right\}\cos\frac{\pi x}{a}\,d\left(\frac{x}{a}\right)$$

$$+\frac{1}{2}\lambda^4\frac{f\pi^2}{16}\frac{4\lambda^2}{(4+\lambda^2)^2}\int_0^1\left\{(a_1+2b_1)\cosh\frac{\pi\lambda x}{a}+b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\right\}\cos\frac{2\pi x}{a}\,d\left(\frac{x}{a}\right)$$

$$+\frac{1}{4}\lambda^8\int_0^1\left\{(a_1+2b_1)\cosh\frac{\pi\lambda x}{a}+b_1\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\right\}^2 d\left(\frac{x}{a}\right)+\frac{1}{8}\left\{\frac{f\pi^2}{16}\frac{\lambda^2}{(1+4\lambda^2)^2}\right\}^2$$

$$+2\lambda^4\frac{f\pi^2}{16}\frac{\lambda^2}{(1+4\lambda^2)^2}\int_0^1\left\{(a_2+2b_2)\cosh\frac{2\pi\lambda x}{a}+b_2\left(\frac{2\pi\lambda x}{a}\right)\sinh\frac{2\pi\lambda x}{a}\right\}\cos\frac{\pi x}{a}\,d\left(\frac{x}{a}\right)$$

$$+4\lambda^8\int_0^1\left\{(a_2+2b_2)\cosh\frac{2\pi\lambda x}{a}+b_2\left(\frac{2\pi\lambda x}{a}\right)\sinh\frac{2\pi\lambda x}{a}\right\}^2 d\left(\frac{x}{a}\right)\Bigg]+\frac{1}{2}\left(\frac{\sigma}{E}\right)^2$$

$$\underline{\underline{\frac{1}{ab}\int_0^a\int_0^b\left(\frac{\tau_{xy}}{E}\right)^2 dx\,dy}} = \left(\frac{a}{R}\right)^4\left(\frac{f}{g}\right)^2\left[\frac{1}{4}\left\{\left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^3}{(1+\lambda^2)^2}\right\}^2 + \frac{1}{4}\left\{\frac{f\pi^2}{16}\frac{2\lambda^3}{(4+\lambda^2)^2}\right\}^2\right.$$

$$+\lambda^4\left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^3}{(1+\lambda^2)^2}\int_0^1\left\{(a_1+b_1)\sinh\frac{\pi\lambda x}{a}+b_1\left(\frac{\pi\lambda x}{a}\right)\cosh\frac{\pi\lambda x}{a}\right\}\sin\frac{\pi x}{a}\,d\left(\frac{x}{a}\right)$$

$$+\lambda^4\left(\frac{f\pi^2}{16}\right)\frac{2\lambda^3}{(4+\lambda^2)^2}\int_0^1\left\{(a_1+b_1)\sinh\frac{\pi\lambda x}{a}+b_1\left(\frac{\pi\lambda x}{a}\right)\cosh\frac{\pi\lambda x}{a}\right\}\sin\frac{2\pi x}{a}\,d\left(\frac{x}{a}\right)$$

$$+\frac{1}{2}\lambda^8\int_0^1\left\{(a_1+b_1)\sinh\frac{\pi\lambda x}{a}+b_1\left(\frac{\pi\lambda x}{a}\right)\cosh\frac{\pi\lambda x}{a}\right\}^2 d\left(\frac{x}{a}\right)+\frac{1}{4}\left\{\frac{f\pi^2}{16}\frac{2\lambda^3}{(1+4\lambda^2)^2}\right\}^2$$

$$+4\lambda^4\frac{f\pi^2}{16}\frac{2\lambda^3}{(1+4\lambda^2)^2}\int_0^1\left\{(a_2+b_2)\sinh\frac{2\pi\lambda x}{a}+b_2\left(\frac{2\pi\lambda x}{a}\right)\cosh\frac{2\pi\lambda x}{a}\right\}\sin\frac{\pi x}{a}\,d\left(\frac{x}{a}\right)$$

$$\left.+\,8\lambda^8\int_0^1\left\{(a_2+b_2)\sinh\frac{2\pi\lambda x}{a}+b_2\left(\frac{2\pi\lambda x}{a}\right)\cosh\frac{2\pi\lambda x}{a}\right\}^2 d\left(\frac{x}{a}\right)\right]$$

$$\int_0^1 \cos\frac{2\pi x}{a}\cosh\frac{\pi\lambda x}{a}\, d\left(\frac{x}{a}\right) = \frac{1}{2\pi}\int_0^{\pi}\left[\cosh(\lambda+2i)\theta + \cosh(\lambda-2i)\theta\right]d\theta$$

$$= \frac{1}{2\pi}\left[\frac{\sinh(\lambda+2i)\pi}{\lambda+2i} + \frac{\sinh(\lambda-2i)\pi}{\lambda+2i}\right] = \frac{\lambda}{\pi}\,\frac{\sinh\lambda\pi}{4+\lambda^2}$$

$$\int_0^1 \cos\frac{2\pi x}{a}\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\, d\left(\frac{x}{a}\right) = \frac{\lambda}{2\pi}\left[\pi\left\{\frac{\cosh(\lambda+2i)\pi}{\lambda+2i} + \frac{\cosh(\lambda-2i)\pi}{\lambda-2i}\right\}\right.$$

$$\left. - \left\{\frac{\sinh(\lambda+2i)\pi}{(\lambda+2i)^2} + \frac{\sinh(\lambda-2i)\pi}{(\lambda-2i)^2}\right\}\right] = \left[\frac{\lambda^2\cosh\lambda\pi}{(4+\lambda^2)} + \frac{\lambda(4-\lambda^2)\sinh\lambda\pi}{\pi\,(4+\lambda^2)^2}\right]$$

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$$\frac{1}{2ab}\int_0^a\int_0^b\left(\frac{\sigma_x}{E}\right)^2 dx\,dy = \left(\frac{a}{R}\right)^4\left(\frac{f}{g}\right)^2\Bigg[\frac{1}{4}\left(\frac{f\pi^2}{16}-1\right)^2+\frac{1}{8}\left\{\left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^4}{(1+\lambda^2)^2}\right\}^2+\frac{1}{8}\left\{\frac{f\pi^2}{16}\frac{\lambda^4}{(4+\lambda^2)^2}\right\}^2$$

$$-\frac{\lambda^4}{2}\left(\frac{f\pi^2}{16}-1\right)\left\{(a_1-b_1)\frac{\sinh\pi\lambda}{\pi\lambda}+b_1\cosh\pi\lambda\right\}$$

$$+\frac{\lambda^4}{2}\left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^4}{(1+\lambda^2)^2}\left\{a_1\frac{\lambda}{\pi}\frac{\sinh\lambda\pi}{1+\lambda^2}+\left[\frac{\lambda^2\cosh\lambda\pi}{1+\lambda^2}+\frac{\lambda(1-\lambda^2)\sinh\lambda\pi}{\pi(1+\lambda^2)^2}\right]b_1\right\}$$

$$-\frac{\lambda^4}{2}\frac{f\pi^2}{16}\frac{\lambda^4}{(4+\lambda^2)^2}\left\{\frac{\lambda}{\pi}\frac{\sinh\lambda\pi}{4+\lambda^2}a_1+\left[\frac{\lambda^2\cosh\lambda\pi}{4+\lambda^2}+\frac{\lambda(4-\lambda^2)\sinh\lambda\pi}{\pi(4+\lambda^2)^2}\right]b_1\right\}$$

$$+\frac{\lambda^8}{4}\left\{a_1^2\left(\frac{1}{2}+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)+\frac{a_1b_1}{2}\left(\cosh 2\pi\lambda-\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)+b_1^2\left(-\frac{\pi^2\lambda^2}{6}+\frac{1}{8}\left[(4\pi^2\lambda^2+2)\frac{\sinh 2\pi\lambda}{2\pi\lambda}-2\cosh 2\pi\lambda\right]\right)\right\}$$

$$+\frac{1}{4}\left(\frac{f\pi^2}{64}\right)^2+\frac{1}{8}\left\{\frac{f\pi^2}{16}\frac{4\lambda^4}{(1+4\lambda^2)^2}\right\}^2-2\lambda^4\frac{f\pi^2}{64}\left\{(a_2-b_2)\frac{\sinh 2\pi\lambda}{2\pi\lambda}+b_2\cosh 2\pi\lambda\right\}$$

$$+2\lambda^4\frac{f\pi^2}{16}\frac{4\lambda^4}{(1+4\lambda^2)^2}\left\{\frac{2\lambda}{\pi}\frac{\sinh 2\lambda\pi}{(1+4\lambda^2)}a_2+\left[\frac{4\lambda^2\cosh 2\lambda\pi}{1+4\lambda^2}+\frac{2\lambda(1-4\lambda^2)\sinh 2\lambda\pi}{\pi(1+4\lambda^2)^2}\right]b_1\right\}$$

$$+4\lambda^8\left\{a_2^2\left(\frac{1}{2}+\frac{\sinh 4\pi\lambda}{4\pi\lambda}\right)+\frac{a_2b_2}{2}\left(\cosh 4\pi\lambda-\frac{\sinh 4\pi\lambda}{4\pi\lambda}\right)+b_2^2\left(-\frac{4\pi^2\lambda^2}{6}+\frac{1}{8}\left[(16\pi^2\lambda^2+2)\frac{\sinh 4\pi\lambda}{4\pi\lambda}-2\cosh 4\pi\lambda\right]\right)\right\}\Bigg]$$

$$\cosh\pi\lambda\cdot a_1+\pi\lambda\sinh\pi\lambda\cdot b_1=\frac{1}{\lambda^4}\left[\left(\frac{f\pi^2}{16}-1\right)-\left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^4}{(1+\lambda^2)^2}+\frac{f\pi^2}{16}\frac{\lambda^4}{(4+\lambda^2)^2}\right]$$

$$\sinh\pi\lambda\cdot a_1+(\sinh\pi\lambda+\pi\lambda\cosh\pi\lambda)\,b_1=0$$

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$$b_1=\frac{1}{\lambda^4}\left[\left(\frac{f\pi^2}{16}-1\right)-\left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^4}{(1+\lambda^2)^2}+\frac{f\pi^2}{16}\frac{\lambda^4}{(4+\lambda^2)^2}\right]\left[-\frac{\frac{\sinh\pi\lambda}{\pi\lambda}}{1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}}\right]$$

$$a_1=\frac{1}{\lambda^4}\left[\left(\frac{f\pi^2}{16}-1\right)-\left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^4}{(1+\lambda^2)^2}+\frac{f\pi^2}{16}\frac{\lambda^4}{(4+\lambda^2)^2}\right]\left[+\frac{\cosh\pi\lambda+\frac{\sinh\pi\lambda}{\pi\lambda}}{1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}}\right]$$

$$a_2=\frac{1}{16\lambda^4}\left(\frac{f\pi^2}{16}\right)\left\{1-\frac{16\lambda^4}{(1+4\lambda^2)^2}\right\}\left\{\frac{\cosh 2\pi\lambda+\frac{\sinh 2\pi\lambda}{2\pi\lambda}}{1+\frac{\sinh 4\pi\lambda}{4\pi\lambda}}\right\}$$

$$b_2=\frac{1}{16\lambda^4}\left(\frac{f\pi^2}{16}\right)\left\{1-\frac{16\lambda^4}{(1+4\lambda^2)^2}\right\}\left\{-\frac{\frac{\sinh 2\pi\lambda}{2\pi\lambda}}{1+\frac{\sinh 4\pi\lambda}{4\pi\lambda}}\right\}$$

$$\frac{1}{2ab}\int_0^a\int_0^b\left(\frac{\sigma_y}{E}\right)^2 dx\,dy = \left(\frac{a}{R}\right)^4\left(\frac{f}{8}\right)^2\Bigg[\frac{1}{4}\left(\frac{f\pi^2}{16}\lambda^2\right)^2+\frac{1}{4}\left(\frac{f\pi^2}{64}\lambda^2\right)^2+\frac{1}{8}\left\{\left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^2}{(1+\lambda^2)^2}\right\}^2+\frac{1}{8}\left\{\frac{f\pi^2}{16}\frac{4\lambda^2}{(4+\lambda^2)^2}\right\}^2$$

$$-\frac{\lambda^4}{2}\left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^2}{(1+\lambda^2)^2}\left\{(a_1+2b_1)\frac{\lambda}{\pi}\frac{\sinh\lambda\pi}{(1+\lambda^2)}+\left[\frac{\lambda^2\cosh\lambda\pi}{1+\lambda^2}+\frac{\lambda(1-\lambda^2)\sinh\lambda\pi}{\pi(1+\lambda^2)^2}\right]b_1\right\}$$

$$+\frac{\lambda^4}{2}\frac{f\pi^2}{16}\frac{4\lambda^2}{(4+\lambda^2)^2}\left\{(a_1+2b_1)\frac{\lambda}{\pi}\frac{\sinh\lambda\pi}{4+\lambda^2}+\left[\frac{\lambda^2\cosh\lambda\pi}{4+\lambda^2}+\frac{\lambda(4-\lambda^2)\sinh\lambda\pi}{\pi(4+\lambda^2)^2}\right]b_1\right\}$$

$$+\frac{\lambda^8}{4}\left\{(a_1+2b_1)^2\left(\frac{1}{2}+\frac{\sinh2\pi\lambda}{2\pi\lambda}\right)+\frac{(a_1+2b_1)b_1}{2}\left(\cosh2\pi\lambda-\frac{\sinh2\pi\lambda}{2\pi\lambda}\right)+b_1^2\left(-\frac{\pi^2\lambda^2}{6}+\frac{1}{8}\left[(4\pi^2\lambda^2+2)\frac{\sinh2\pi\lambda}{2\pi\lambda}-2\cosh2\pi\lambda\right]\right)\right\}$$

$$+\frac{1}{8}\left\{\frac{f\pi^2}{16}\frac{\lambda^2}{(1+4\lambda^2)^2}\right\}^2-2\lambda^4\frac{f\pi^2}{16}\frac{\lambda^2}{(1+4\lambda^2)^2}\left\{(a_2+2b_2)\frac{2\lambda}{\pi}\frac{\sinh2\pi\lambda}{(1+4\lambda^2)}+\left[\frac{4\lambda^2\cosh2\pi\lambda}{1+4\lambda^2}+\frac{2\lambda(1-4\lambda^2)\sinh2\lambda\pi}{\pi(1+4\lambda^2)^2}\right]b_2\right\}$$

$$+4\lambda^8\left\{(a_2+2b_2)^2\left(\frac{1}{2}+\frac{\sinh4\pi\lambda}{4\pi\lambda}\right)+\frac{b_2(a_2+2b_2)}{2}\left(\cosh4\pi\lambda-\frac{\sinh4\pi\lambda}{4\pi\lambda}\right)+b_2^2\left(-\frac{4\pi^2\lambda^2}{6}+\frac{1}{8}\left[(16\pi^2\lambda^2+2)\frac{\sinh4\pi\lambda}{4\pi\lambda}-2\cosh4\pi\lambda\right]\right)\right\}\Bigg]$$

$$+\frac{1}{2}\left(\frac{\overline{\sigma}}{E}\right)^2$$

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$$\frac{1}{ab}\int_0^a\int_0^b\left(\frac{\tau_{xy}}{E}\right)^2 dx\,dy = \left(\frac{a}{R}\right)^4\left(\frac{f}{8}\right)^2\Bigg[\frac{1}{4}\left\{\left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^3}{(1+\lambda^2)^2}\right\}^2+\frac{1}{4}\left\{\frac{f\pi^2}{16}\frac{2\lambda^3}{(4+\lambda^2)^2}\right\}^2$$

$$+\lambda^4\left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^3}{(1+\lambda^2)^2}\left\{(a_1+b_1)\frac{1}{\pi}\frac{\sinh\lambda\pi}{(1+\lambda^2)}+\left[\frac{\lambda\cosh\lambda\pi}{1+\lambda^2}-\frac{2\lambda^2\sinh\lambda\pi}{\pi(1+\lambda^2)^2}\right]b_1\right\}$$

$$-\lambda^4\left(\frac{f\pi^2}{16}\right)\frac{2\lambda^3}{(4+\lambda^2)^2}\left\{(a_1+b_1)\frac{2}{\pi}\frac{\sinh\lambda\pi}{(4+\lambda^2)}+\left[\frac{2\lambda\cosh\lambda\pi}{4+\lambda^2}-\frac{4\lambda^2\sinh\lambda\pi}{\pi(4+\lambda^2)^2}\right]b_1\right\}$$

$$+\frac{\lambda^8}{2}\left\{(a_1+b_1)^2\left(-\frac{1}{2}+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)+\frac{b_1(a_1+b_1)}{2}\left(\cosh 2\pi\lambda-\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)+b_1^2\left(\frac{\pi^2\lambda^2}{6}+\frac{1}{8}\left[(4\pi^2\lambda^2+2)\frac{\sinh 2\pi\lambda}{2\pi\lambda}-2\cosh 2\pi\lambda\right]\right)\right\}$$

$$+\frac{1}{4}\left\{\frac{f\pi^2}{16}\frac{2\lambda^3}{(1+4\lambda^2)^2}\right\}^2+4\lambda^4\frac{f\pi^2}{16}\frac{2\lambda^3}{(1+4\lambda^2)^2}\left\{(a_2+b_2)\frac{1}{\pi}\frac{\sinh 2\pi\lambda}{(1+4\lambda^2)}+\left[\frac{2\lambda\cosh 2\lambda\pi}{1+4\lambda^2}-\frac{8\lambda^2\sinh 2\pi\lambda}{\pi(1+4\lambda^2)^2}\right]b_2\right\}$$

$$+8\lambda^8\left\{(a_2+b_2)^2\left(-\frac{1}{2}+\frac{\sinh 4\pi\lambda}{4\pi\lambda}\right)+\frac{b_2(a_2+b_2)}{2}\left(\cosh 4\pi\lambda-\frac{\sinh 4\pi\lambda}{4\pi\lambda}\right)+\right.$$

$$\left.+b_2^2\left(\frac{4\pi^2\lambda^2}{6}+\frac{1}{8}\left[(16\pi^2\lambda^2+2)\frac{\sinh 4\pi\lambda}{4\pi\lambda}-2\cosh 4\pi\lambda\right]\right)\right\}\Bigg]$$

Terms independent upon the a's & b's

$$\boxed{\frac{1}{4}\left(\frac{f\pi^2}{16}-1\right)^2 + \lambda^6 \frac{17}{16\times 1024}\left(f\pi^2\right)^2 + \frac{\lambda^4}{(1+\lambda^2)^2}\frac{1}{8}\left(\frac{f\pi^2}{8}-1\right)^2 + \frac{\lambda^4}{(4+\lambda^2)^2}\frac{1}{8}\left(\frac{f\pi^2}{16}\right)^2 + \frac{1}{4}\left(\frac{f\pi^2}{64}\right)^2 + \frac{\lambda^4}{(1+4\lambda^2)^2}\frac{1}{8}\left(\frac{f\pi^2}{16}\right)^2}$$

Terms linear in a_1, b_1.

$$-\frac{\lambda^4}{2}\left(\frac{f\pi^2}{16}-1\right)\left\{(a_1-b_1)\frac{\sinh \pi\lambda}{\pi\lambda} + b_1\cosh\pi\lambda\right\} + \frac{\lambda^4}{2}\left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^3}{(1+\lambda^2)^3}\frac{\sinh\lambda\pi}{\pi}\left\{\lambda^2 a_1 - (a_1+2b_1) + 2(a_1+b_1)\right\}$$

$$+\frac{\lambda^4}{2}\left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^4}{(1+\lambda^2)^3}\cosh\lambda\pi\left\{\lambda^2 b_1 - b_1 + 2b_1\right\} + \frac{\lambda^4}{2}\left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^3}{(1+\lambda^2)^4}\frac{\sinh\lambda\pi}{\pi}\left\{\lambda^2(1-\lambda^2)b_1 - (1-\lambda^2)b_1 - 4\lambda^2 b_1\right\}$$

$$-\frac{\lambda^4}{2}\frac{f\pi^2}{16}\frac{\lambda^3}{(4+\lambda^2)^3}\frac{\sinh\lambda\pi}{\pi}\left\{\lambda^2 a_1 - 4(a_1+2b_1) + 8(a_1+b_1)\right\}$$

$$-\frac{\lambda^4}{2}\frac{f\pi^2}{16}\frac{\lambda^4}{(4+2\lambda^2)^3}\cosh\lambda\pi\left\{\lambda^2 b_1 - 4b_1 + 8b_1\right\} - \frac{\lambda^4}{2}\frac{f\pi^2}{16}\frac{\lambda^3}{(4+\lambda^2)^4}\frac{\sinh\lambda\pi}{\pi}\left\{\lambda^2(4-\lambda^2)b_1 - 4(4-\lambda^2)b_1 - 16\lambda^2 b_1\right\}$$

$$= \frac{\lambda^4}{2}\Big[-\left(\frac{f\pi^2}{16}-1\right)\left\{(a_1-b_1)\frac{\sinh \pi\lambda}{\pi\lambda} + b_1 \cosh \pi\lambda\right\} + \left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^4}{(1+\lambda^2)^2}\left\{\frac{\sinh \lambda\pi}{\lambda\pi}(a_1-b_1) + b_1 \cosh \pi\lambda\right\}$$

$$- \frac{f\pi^2}{16}\frac{\lambda^4}{(4+\lambda^2)^2}\left\{\frac{\sinh \lambda\pi}{\lambda\pi}(a_1-b_1) + b_1 \cosh \pi\lambda\right\}$$

The terms linear in a_2, b_2

$$2\lambda^4\left[-\frac{f\pi^2}{64}\left\{(a_2-b_2)\frac{\sinh 2\pi\lambda}{2\pi\lambda} + b_2 \cosh 2\pi\lambda\right\} + \frac{f\pi^2}{16}\frac{4\lambda^4}{(1+4\lambda^2)^2}\left\{\frac{\sinh 2\pi\lambda}{2\pi\lambda}(a_2-b_1) + b_1 \cosh 2\pi\lambda\right\}\right]$$

The terms linear in a_1, b_1,

$$-\left[\left(\frac{f\pi^2}{16}-1\right) - \left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^4}{(1+\lambda^2)^2} + \frac{f\pi^2}{16}\frac{\lambda^4}{(4+\lambda^2)^2}\right]^2 \frac{\left(\frac{\sinh \pi\lambda}{\pi\lambda}\right)^2}{1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}}$$

The terms linear in a_2, b_2

$$-\frac{1}{16}\left[\frac{f\pi^2}{16}\left\{1-\frac{16\lambda^4}{(1+4\lambda^2)^2}\right\}\right]^2 \frac{\left(\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2}{1+\frac{\sinh 4\pi\lambda}{4\pi\lambda}}$$

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Terms of containing $a_1, b_1,$

$$\frac{1}{4}\left[\left(\frac{\pi^2}{16}-1\right)-\left(\frac{\pi^2}{8}-1\right)\frac{\lambda^4}{(1+\lambda^2)^2}+\frac{\pi^2}{16}\cdot\frac{\lambda^4}{(4+\lambda^2)^2}\right]^2\frac{\frac{\sinh\pi\lambda}{\pi\lambda}\left\{(2\pi^2\lambda^2-1)\frac{\sinh\pi\lambda}{\pi\lambda}\frac{\sinh2\pi\lambda}{2\pi\lambda}+2\left(\cosh\pi\lambda-\frac{\sinh\pi\lambda}{\pi\lambda}\right)\right\}}{\left(1+\frac{\sinh2\pi\lambda}{2\pi\lambda}\right)^2}$$

Terms of containing a_2, b_2

$$\frac{1}{64}\left[\left(\frac{\pi^2}{16}\right)^2\left\{1-\frac{16\lambda^4}{(1+4\lambda^2)^2}\right\}^2\right]\frac{\frac{\sinh2\pi\lambda}{2\pi\lambda}\left\{(8\pi^2\lambda^2-1)\frac{\sinh2\pi\lambda}{2\pi\lambda}\frac{\sinh4\pi\lambda}{4\pi\lambda}+2\left(\cosh2\pi\lambda-\frac{\sinh2\pi\lambda}{2\pi\lambda}\right)\right\}}{\left(1+\frac{\sinh4\pi\lambda}{4\pi\lambda}\right)^2}$$

$$H_1(\lambda)=\frac{17}{16384}(1+\lambda^4)+\frac{1}{512}\frac{\lambda^4}{(1+\lambda^2)^2}+\frac{1}{2048}\frac{\lambda^4}{(4+\lambda^2)^2}+\frac{1}{2048}\frac{\lambda^4}{(1+4\lambda^2)^2}$$

$$+\frac{1}{1024}\left[1-2\frac{\lambda^4}{(1+\lambda^2)^2}+\frac{\lambda^4}{(4+\lambda^2)^2}\right]^2\frac{\frac{\sinh\pi\lambda}{\pi\lambda}\left\{(2\pi^2\lambda^2-1)\frac{\sinh\pi\lambda}{\pi\lambda}\frac{\sinh2\pi\lambda}{2\pi\lambda}+2\left(\cosh\pi\lambda-\frac{\sinh\pi\lambda}{\pi\lambda}\right)\right\}}{\left(1+\frac{\sinh2\pi\lambda}{2\pi\lambda}\right)^2}$$

$$+\frac{1}{16384}\left\{1-\frac{16\lambda^4}{(1+4\lambda^2)^2}\right\}^2\frac{\frac{\sinh2\pi\lambda}{2\pi\lambda}\left\{(8\pi^2\lambda^2-1)\frac{\sinh2\pi\lambda}{2\pi\lambda}\frac{\sinh4\pi\lambda}{4\pi\lambda}+2\left(\cosh2\pi\lambda-\frac{\sinh2\pi\lambda}{2\pi\lambda}\right)\right\}}{\left(1+\frac{\sinh4\pi\lambda}{4\pi\lambda}\right)^2}$$

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$$H_2(\lambda) = \frac{1}{32} + \frac{1}{32}\frac{\lambda^4}{(1+\lambda^2)^2} + \frac{1}{32}\left[1 - 2\frac{\lambda^4}{(1+\lambda^2)^2} + \frac{\lambda^4}{(4+\lambda^2)^2}\right]\left[1 - \frac{\lambda^4}{(1+\lambda^2)^2}\right] \frac{\frac{\sinh\pi\lambda}{\pi\lambda}\left\{(2\pi^2\lambda^2-1)\frac{\sinh\pi\lambda}{\pi\lambda}\frac{\sinh 2\pi\lambda}{2\pi\lambda} + 2\left(\cosh\pi\lambda - \frac{\sinh\pi\lambda}{\pi\lambda}\right)\right\}}{\left(1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2}$$

$$H_3(\lambda) = \frac{1}{4} + \frac{1}{8}\frac{\lambda^4}{(1+\lambda^2)^2} + \frac{1}{4}\left[1 - \frac{\lambda^4}{(1+\lambda^2)^2}\right]^2 \frac{\frac{\sinh\pi\lambda}{\pi\lambda}\left\{(2\pi^2\lambda^2-1)\frac{\sinh\pi\lambda}{\pi\lambda}\frac{\sinh 2\pi\lambda}{2\pi\lambda} + 2\left(\cosh\pi\lambda - \frac{\sinh\pi\lambda}{\pi\lambda}\right)\right\}}{\left(1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)^2}$$

$$\text{Bending energy} = \frac{1}{24}\left(\frac{t}{R}\right)^2\left(\frac{f}{\delta}\right)^2\pi^4\left\{\frac{3}{4} + \frac{3}{4}\lambda^4 + \frac{1}{2}\lambda^2\right\} = \left(\frac{t}{R}\right)^2\left(\frac{f}{\delta}\right)^2\pi^4\left\{\frac{1}{32}(1+\lambda^4) + \frac{1}{48}\lambda^2\right\}$$

$$\boxed{\lambda^2 K = 2\left[\frac{512}{9}H_1H_3\left(\frac{\delta}{t}\right)^2 + H_3\left\{\frac{2}{9}(1+\lambda^4) + \frac{4}{27}\lambda^2\right\}\right]^{\frac{1}{2}} - 8H_2\left(\frac{\delta}{t}\right)}$$

$$\gamma^2 = \pi^2\left[\frac{8H_1}{H_3}\left(\frac{\delta}{t}\right)^2 + \frac{1}{H_3}\left\{\frac{1}{32}(1+\lambda^4) + \frac{1}{48}\lambda^2\right\}\right]^{\frac{1}{2}}$$

$$\boxed{\left(\frac{a}{R}\right) = \pi\left(\frac{t}{R}\right)^{\frac{1}{2}}\left[\frac{8H_1}{H_3}\left(\frac{\delta}{t}\right)^2 + \frac{1}{H_3}\left\{\frac{1}{32}(1+\lambda^4) + \frac{1}{48}\lambda^2\right\}\right]^{\frac{1}{4}}}$$

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λ	$\pi\lambda$	$\log_{10}(e^{\pi\lambda})$	$e^{\pi\lambda}$	$e^{-\pi\lambda}$	$\sinh\pi\lambda$	$\sinh\pi\lambda/\pi\lambda$	$\cosh\pi\lambda$	$\cosh\pi\lambda - \sinh\pi\lambda/\pi\lambda$
0.05	0.1570796	0.0682188	1.170089	0.854636	0.1577265	1.004118	1.012363	0.008245
0.10	0.3141593	0.1364376	1.369108	0.730403	0.3193525	1.016530	1.049756	0.033226
0.15	0.4712389	0.2046565	1.601978	0.624228	0.4888750	1.037425	1.113103	0.075678
0.20	0.6283185	0.2728753	1.874456	0.533488	0.6704840	1.067108	1.203972	0.136864
0.30	0.9424778	0.4093129	2.566333	0.389661	1.0883360	1.154760	1.477997	0.323237
0.40	1.2566371	0.5457506	3.513586	0.284610	1.614488	1.284769	1.719098	0.434329
0.50	1.5707964	0.6821882	4.810478	0.207880	2.301299	1.465053	2.509179	1.044126
0.60	1.8849556	0.8186258	6.586065	0.151836	3.217115	1.706732	3.368951	1.662219
0.80	2.5132742	1.0915011	12.34529	0.081003	6.132144	2.439903	6.213146	3.77243
1.00	3.1415927	1.3643764	23.14070	0.043214	11.54874	3.676078	11.59196	7.91588
1.20	3.7699112	1.6372517	43.37623	0.023054	21.67659	5.749894	21.69964	15.94975
1.40	4.3982298	1.9101270	81.30683	0.012299	40.64727	9.241734	40.65907	31.41734
1.60	5.0265483	2.1830022	152.4060	0.006561	76.19972	15.15945	76.20628	61.04683
1.80	5.6548669	2.4558775	285.6785	0.003500	142.8375	25.25921	142.8410	122.5818
2.00	6.2831854	2.7287528	535.4917	0.001867	267.7449	42.61293	267.7468	225.1339
2.20	6.9115039	3.0016281	1003.756	0.000996	501.8775	72.61480	501.8785	429.2637

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λ	$e^{2\pi\lambda}$	$e^{-2\pi\lambda}$	$\sinh 4\pi\lambda/2$	$\sinh 2\pi\lambda/2\pi\lambda$	$\cosh 2\pi\lambda$	$\cosh 2\pi\lambda - \sinh 2\pi\lambda/2\pi\lambda$	$\sinh 4\pi\lambda$	$\sinh 4\pi\lambda/4\pi\lambda$
0.05			0.3193525/2	1.016530	1.049756	0.033226	0.6704840	1.067108
0.10			0.6704840/2	1.067108	1.203972	0.136864	1.614488	1.284769
0.15			1.0883360/2	1.154760	1.477997	0.323237	3.217115	1.706732
0.20			1.614488/2	1.284769	1.719098	0.434329	6.132144	2.439903
0.30			3.217115/2	1.706732	3.368951	1.662219	21.67659	5.749894
0.40			—	2.439903	6.213146	3.77243	76.19972	15.15945
0.50			—	3.676078	11.59196	7.91588	267.7449	42.61293
0.60			—	5.749894	21.69764	15.94975	940.7484	124.7707
0.80			—	15.15945	76.20628	61.04683	11613.79	1155.245
1.00			—	42.61293	267.7468	225.1339	143375.7	11409.474
1.20	1881.4973	0.0005	470.3742	124.7707	940.7489	815.9782	1770016	117377.90
1.40	6610.8006	0.0001	1652.7001	375.7648	3305.400	2929.635	21851340	1242053
1.60	23227.589		5806.8973	1155.245	11613.79	10458.54	269760300	13416770
1.80	81612.205		20403.0513	3608.051	40806.10	37198.05	3330276000	147230500
2.00	286751.36		71687.840	11409.474	143375.7	131966.2	41113180000	1635841000
2.20	1007526.1		251881.53	36443.81	503763.1	467319.3	507554500000	18359050000

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λ	$\sinh 2\pi\lambda$	$\dfrac{\frac{\sinh \pi\lambda}{\pi\lambda}}{1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}}$	$\dfrac{\sinh 2\pi\lambda/2\pi\lambda}{1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}}$	$\dfrac{2\left(\cosh\pi\lambda-\frac{\sinh\pi\lambda}{\pi\lambda}\right)}{1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}}$	$2\pi^2\lambda^2-1$	I		
0.05	0.3193525	0.4979435	0.5040986	0.008177	−0.9506520	−0.2355366		
0.10	0.6704840	0.4917643	0.5162323	0.032147	−0.8026089	−0.1913133		
0.15	1.0883360	0.4814573	0.5359112	0.070243	−0.5558678	−0.1149728		
0.20	1.614488	0.4670529	0.5623190	0.119806	−0.2104316	−0.0030193		
0.30	3.217115	0.4266252	0.6305508	0.238839	+0.7765288	+0.3431162		
0.40	6.132144	0.3734899	0.7092941	0.352524	+2.158273	+0.828891		
0.50	11.54874	0.3133081	0.7861456	0.446582	+3.934802	+1.559795		
0.60	21.67659	0.2528532	0.8518495	0.492517	+6.106115	+2.369252		
0.70	76.19972	0.1509892	0.9381167	0.461901	+11.63309	+4.090907		
1.00	267.7449	0.084288719	0.9770210	0.363006	+18.73921	+5.703843		
1.20	940.7483	0.04571728	0.9920490	0.253632	+27.42446	+7.163331		
1.40	3305.400	0.02452919	0.9973458	0.166774	+37.68885	+8.525184		
1.60	11613.79	0.01311093	0.9991351	0.105595	+49.53237	+9.837651		
1.80	40808.10	0.006998851	0.9997229	0.067930	+62.95504	+11.12693		
2.00	143375.7	0.003734545	0.9999127	0.039461	+77.95684	+12.40511		
2.20	503763.1	0.001992459	0.9999726	0.023557	+94.53727	+13.67759		

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λ	$\dfrac{\frac{\sinh 2\pi\lambda}{2\pi\lambda}}{1+\frac{\sinh 4\pi\lambda}{4\pi\lambda}}$	$\dfrac{\frac{\sinh 4\pi\lambda}{4\pi\lambda}}{1+\frac{\sinh 4\pi\lambda}{4\pi\lambda}}$	$\dfrac{2\left(\cosh 2\pi\lambda-\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)}{1+\frac{\sinh 4\pi\lambda}{4\pi\lambda}}$	$8\pi^2\lambda^2-1$	II	λ^4	$1+\lambda^2$	$4+\lambda^2$	$1+4\lambda^2$
0.05	0.4917643	0.5162323	0.032147	−0.8026079	−0.1913133	0.00000625	1.0025	4.0025	1.010
0.10	0.4670529	0.5623190	0.119806	−0.2104317	−0.0030193	0.0001	1.01	4.01	1.040
0.15	0.4266252	0.6305508	0.238839	+0.7765288	+0.3431162	0.00050625	1.0225	4.0225	1.090
0.20	0.3734899	0.7092941	0.252524	+2.158273	+0.828891	0.0016	1.04	4.04	1.160
0.30	0.2528532	0.8518495	0.492517	+6.106115	+2.369252	0.0081	1.09	4.09	1.360
0.40	0.1569892	0.9381167	0.466901	+11.63309	+4.090907	0.0256	1.16	4.16	1.64
0.50	0.084288719	0.9770710	0.363006	+18.73921	+5.703843	0.0625	1.25	4.25	2.00
0.60	0.04571728	0.9920490	0.253632	+27.42446	+7.163331	0.1296	1.36	4.36	2.44
0.80	0.01311093	0.9991351	0.105595	+49.53237	+9.837651	0.4096	1.64	4.64	3.56
1.00	0.003734545	0.9999127	0.039461	+77.95684	+12.40511	1.0000	2.00	5.00	5.00
1.20	0.001062973	0.9999915	0.0139033	+112.6978	+14.94676	2.0736	2.44	5.44	6.76
1.40	0.0003025350	0.9999992	0.004717	+153.7554	+17.47921	3.8416	2.96	5.96	8.84
1.60	0.0000861045	1	0.001559	+201.1295	+20.00672	6.5536	3.56	6.56	11.24
1.80	0.00002450614	1	0.000505	+254.8201	+22.53104	10.4976	4.24	7.24	13.96
2.00	0.000006974684	1	—	+314.8243	+25.05316	16.0000	5.00	8.00	17.00
2.20	0.000001985060	1	—	+381.1511	+27.57367	23.4256	5.84	8.84	20.36

	①	②	③	④	⑤	⑥	⑦	⑧
λ	$\frac{17}{16384}(1+\lambda^4)$	$\frac{\lambda^4}{(1+\lambda^2)^2}$	$\frac{\lambda^4}{(4+\lambda^2)^2}$	$\frac{\lambda^4}{(1+4\lambda^2)^2}$	$\frac{1}{512}$ ②	③+④	$\frac{1}{2048}$ ⑥	①+⑤+⑦
0.05	0.001037604	0.00000622	0.00000039	0.00000613	0.000000012	0.00000652	0.000000003	0.001037619
0.10	0.001037702	0.00009803	0.00000122	0.00009246	0.000000191	0.00009868	0.000000048	0.001037941
0.15	0.001038123	0.00048422	0.00003129	000042610	0.000000946	0.00045739	0.000000223	0.001039292
0.20	0.001039258	0.00147929	0.00009803	0.00118906	0.000002889	0.00128709	0.000000628	0.001042775
0.30	0.001046003	0.00681761	0.00048422	0.00437933	0.000013316	0.00486355	0.000002375	0.001061694
0.40	0.001064161	0.01902497	0.00147929	0.00951814	0.000037158	0.01099743	0.000005370	0.001106689
0.50	0.001102448	0.04000000	0.00346021	0.01562500	0.000078125	0.01908521	0.000009319	0.001189892
0.60	0.001172071	0.07006920	0.00681761	0.02176834	0.000136854	0.02858595	0.000013958	0.001322883
0.80	0.001462598	0.15229030	0.01902497	0.03231915	0.000297442	0.05134412	0.000025070	0.001785110
1.00	0.002075196	0.2500000	0.0400000	0.0400000	0.000488281	0.08000000	0.000039062	0.002602539
1.20	0.003189161	0.34829347	0.07006920	0.04537656	0.000680261	0.11544576	0.000056370	0.003925792
1.40	0.005023634	0.43845873	0.10814828	0.04715952	0.000856365	0.15730780	0.000076810	0.005956809
1.60	0.007837600	0.51710643	0.15229030	005187371	0.001009973	0.20416401	0.000099689	0.008947262
1.80	0.011929887	0.58392616	0.20026861	0.05386655	0.001140482	0.25413516	0.000124089	0.01319446
2.00	0.017639166	0.640000	0.25000000	0.05536332	0.001250000	0.30536332	0.000149103	0.01903827
2.20	0.025343954	0.68217452	0.29976864	0.05651284	0.001332372	0.35627992	0.000173965	0.02685029

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	9	10	11	12	13	14	15	16
λ	$1-2\times②+③$	$1-16\times④$	$\frac{1}{1024}⑨^2\times I$	$\frac{1}{16384}⑩^2\times II$	H_1	$1-②$	$⑨\times⑭\times I$	H_2
0.05	0.9999880	0.9999019	−0.000230011	−0.000001167	+0.000806441	0.9999738	−0.2355323	+0.02388981
0.10	0.9998102	0.9985206	−0.000186758	−0.000000018	+0.000851165	0.9999019	−0.1912582	+0.02527624
0.15	0.9990629	0.9931824	−0.000112068	+0.000002066	+0.000929290	0.9995158	−0.1148094	+0.02767734
0.20	0.9971395	0.9809750	−0.000002932	+0.000004868	+0.001044711	0.9985207	−0.0030062	+0.03120228
0.30	0.9868490	0.9299072	+0.000326319	+0.000012505	+0.001400518	0.9931824	+0.3362954	+0.04197228
0.40	0.9634294	0.8477098	+0.000751341	+0.000017943	+0.001875973	0.9809750	+0.7833850	+0.05632531
0.50	0.9234602	0.7500000	+0.001298984	+0.000019582	+0.002508458	0.960000	+1.3827922	+0.07571226
0.60	0.8666792	0.6517066	+0.001738114	+0.000018570	+0.003079567	0.9299308	+1.9095027	+0.09311162
0.80	0.7144444	0.4828936	+0.002039185	+0.000014001	+0.003838296	0.8477097	+2.4776226	+0.1134348
1.00	0.5400000	0.3600000	+0.001624259	+0.000009812	+0.004236610	0.7500000	+2.3100564	+0.1112518
1.20	0.3734823	0.2739750	+0.000975788	+0.000006847	+0.004908427	0.6517065	+1.7435605	+0.09662044
1.40	0.2312308	0.2134477	+0.000445138	+0.000004861	+0.006406808	0.5615413	+1.1069576	+0.07954426
1.60	0.1180774	0.1700206	+0.000133945	+0.000001527	+0.009082734	0.4828936	+0.5609313	+0.06493867
1.80	0.0324153	0.1381352	+0.000011417	+0.000001620	+0.01320750	0.4160734	+0.1500705	+0.05418741
2.00	−0.030000	0.1141819	+0.000010903	+0.000001990	+0.01905116	0.360000	−0.1339752	+0.04706328
2.20	−0.0645804	0.0958195	+0.000055707	+0.000001544	+0.02690754	0.3178255	−0.2807366	+0.04379493

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λ	(17) $2\times(14)^2\times I$	(18) H_3	(19) $\frac{2}{9}(1+\lambda^4)+\frac{4}{27}\lambda^2$	(20) $(29)/\lambda^4$	(21) $(18)\times(20)$	(22) K_0	(23) H_1/λ^4	(24) $\frac{512}{9}(23)H_3$
0.05	-0.4710674	+0.1911174	0.2225940	35615.04	6806.654		129.0306	1402.880
0.10	-0.3825515	+0.2021933	0.2237259	2237.259	452.3588		8.51165	97.90570
0.15	-0.2297230	+0.2213452	0.2256681	445.7641	98.66774		1.835635	23.11447
0.20	-0.0060207	+0.2494323	0.2285037	142.8148	35.62262		0.6529444	9.265233
0.3	+0.6769074	+0.3354856	0.2373556	29.30316	9.830202		0.1729035	3.299736
0.4	+1.5953034	+0.4517911	0.2516148	9.828703	4.440520		0.07328020	1.883440
0.5	+2.8750141	+0.6143768	0.2731481	4.370370	2.685054		0.04013533	1.402778
0.6	+4.0977223	+0.7709739	0.3043556	2.348423	1.810573		0.02376209	1.042202
0.8	+5.8954477	+1.0059709	0.4080593	0.9962385	1.002187		0.009370840	0.5362798
1.0	+6.4168234	+1.0833529	0.5925925	0.5925925	0.6419868	1.602482	0.004236610	0.2611054
1.2	+6.0844392	+1.0541416	0.8963555	0.4322702	0.4556740	1.350072	0.002367104	0.1419527
1.4	+5.3764695	+0.9768660	1.3612815	0.3556543	0.3474266	1.178858	0.001667745	0.0926813
1.6	+4.5880099	+0.8881395	2.0578370	0.31400100	0.2788767	1.056176	0.001385915	0.0700238
1.8	+3.8525226	+0.8045562	3.0350222	0.2891158	0.2326099	0.964592	0.001258145	0.0575857
2.0	+3.2154045	+0.7319256	4.0269360	0.2516835	0.1842136	0.858402	0.001190698	0.0495788
2.2	+2.7632302	+0.6806756	6.1449481	0.2623176	0.1785532	0.845112	0.00114838	0.0444686

λ	$8H_2/\lambda^2$
0.05	76.44739
0.10	20.22099
0.15	9.840832
0.20	6.240456
0.30	3.730869
0.40	2.816266
0.50	2.422792
0.60	2.069147
0.80	1.417935
1.00	0.8900144
1.20	0.5367802
1.40	0.3246704
1.60	0.2029333
1.80	0.1337961
2.00	0.0941265
2.20	0.0723883

$$\sigma_{y\,y=b} = -\sigma + E\left(\frac{q}{R}\right)^2 \frac{f\lambda^2}{8}\Bigg[\frac{f\pi^2}{16}\cos\frac{\pi x}{a} + \frac{f\pi^2}{64}\cos\frac{2\pi x}{a} - \left(\frac{f\pi^2}{8}-1\right)\frac{\cos\frac{\pi x}{a}}{(1+\lambda^2)^2} - \frac{f\pi^2}{16}\frac{4}{(4+\lambda^2)^2}\cos\frac{2\pi x}{a}$$

$$-\frac{1}{\lambda^2}\left\{\left(\frac{f\pi^2}{16}-1\right)-\left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^4}{(1+\lambda^2)^2}+\frac{f\pi^2}{16}\frac{\lambda^4}{(4+\lambda^2)^2}\right\}\frac{\left\{\left(\cosh\pi\lambda-\frac{\sinh\pi\lambda}{\pi\lambda}\right)\cosh\frac{\pi\lambda x}{a}-\left(\frac{\sinh\pi\lambda}{\pi\lambda}\right)\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\right\}}{1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}}$$

$$+\frac{f\pi^2}{16}\frac{1}{(1+4\lambda^2)^2}\cos\frac{\pi x}{a}+\frac{1}{4\lambda^2}\left(\frac{f\pi^2}{16}\right)\left\{1-\frac{16\lambda^4}{(1+4\lambda^2)^2}\right\}\frac{\left\{\left(\cosh 2\pi\lambda-\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)\cosh\frac{2\pi\lambda x}{a}-\left(\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)\left(\frac{2\pi\lambda x}{a}\right)\sinh\frac{2\pi\lambda x}{a}\right\}}{1+\frac{\sinh 4\pi\lambda}{4\pi\lambda}}\Bigg]$$

$$= -\sigma + E\left(\frac{q}{R}\right)^2\frac{f\lambda^2}{8}\Bigg\{\left[\frac{f\pi^2}{16}\left\{1-2\frac{1}{(1+\lambda^2)^2}+\frac{1}{(1+4\lambda^2)^2}\right\}+\frac{1}{(1+\lambda^2)^2}\right]\cos\frac{\pi x}{a}+$$

$$+\frac{f\pi^2}{64}\left\{1-\frac{16}{(4+\lambda^2)^2}\right\}\cos\frac{2\pi x}{a}$$

$$-\frac{1}{\lambda^2}\left[\frac{f\pi^2}{16}\left\{1-2\frac{\lambda^4}{(1+\lambda^2)^2}+\frac{\lambda^4}{(4+\lambda^2)^2}\right\}-\left\{1-\frac{\lambda^4}{(1+\lambda^2)^2}\right\}\right]\frac{\left\{\left(\cosh\pi\lambda-\frac{\sinh\pi\lambda}{\pi\lambda}\right)\cosh\frac{\pi\lambda x}{a}-\left(\frac{\sinh\pi\lambda}{\pi\lambda}\right)\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a}\right\}}{1+\frac{\sinh 2\pi\lambda}{2\pi\lambda}}$$

$$+\frac{1}{\lambda^2}\frac{f\pi^2}{64}\left\{1-\frac{16\lambda^4}{(1+4\lambda^2)^2}\right\}\frac{\left\{\left(\cosh 2\pi\lambda-\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)\cosh\frac{2\pi\lambda x}{a}-\left(\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)\left(\frac{2\pi\lambda x}{a}\right)\sinh\frac{2\pi\lambda x}{a}\right\}}{1+\frac{\sinh 4\pi\lambda}{4\pi\lambda}}\Bigg\}$$

$$\frac{1}{\left(\frac{t}{b}\right)}\left(\frac{\sigma_y}{E}\right)_{y=b} = -K + \frac{\lambda^2}{4}\left(\frac{\delta}{t}\right) \left\{ \left[\frac{\pi^2}{8} \frac{\left(\frac{\delta}{t}\right)}{\gamma^2} \left\{ 1 - \frac{2}{(1+\lambda^2)^2} + \frac{1}{(1+4\lambda^2)^2} \right\} + \frac{1}{(1+\lambda^2)^2} \right] \cos\frac{\pi x}{a} + \right.$$

$$+ \frac{\pi^2}{32} \frac{\left(\frac{\delta}{t}\right)}{\gamma^2} \left\{ 1 - \frac{16}{(4+\lambda^2)^2} \right\} \cos\frac{2\pi x}{a} -$$

$$- \frac{1}{\lambda^2}\left[\frac{\pi^2}{8} \frac{\left(\frac{\delta}{t}\right)}{\gamma^2} \left\{ 1 - 2\frac{\lambda^4}{(1+\lambda^2)^2} + \frac{\lambda^4}{(4+\lambda^2)^2} \right\} - \left\{ 1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right\} \right] \frac{\left\{ \left(\cosh\pi\lambda - \frac{\sinh\pi\lambda}{\pi\lambda}\right)\cosh\frac{\pi\lambda x}{a} - \left(\frac{\sinh\pi\lambda}{\pi\lambda}\right)\left(\frac{\pi\lambda x}{a}\right)\sinh\frac{\pi\lambda x}{a} \right\}}{1 + \frac{\sinh 2\pi\lambda}{2\pi\lambda}}$$

$$\left. + \frac{1}{\lambda^2} \frac{\pi^2}{32} \frac{\left(\frac{\delta}{t}\right)}{\gamma^2} \left\{ 1 - \frac{16\lambda^4}{(1+4\lambda^2)^2} \right\} \frac{\left\{ \left(\cosh 2\pi\lambda - \frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)\cosh\frac{2\pi\lambda x}{a} - \left(\frac{\sinh 2\pi\lambda}{2\pi\lambda}\right)\left(\frac{2\pi\lambda x}{a}\right)\sinh\frac{2\pi\lambda x}{a} \right\}}{1 + \frac{\sinh 4\pi\lambda}{4\pi\lambda}} \right\}$$

$$\frac{1}{\left(\frac{t}{b}\right)}\left(\frac{\sigma_y}{E}\right)_{y=b,\,x=0} = -K + \frac{\lambda^2}{4}\left(\frac{\delta}{t}\right) \left\{ \frac{\pi^2}{8} \frac{\left(\frac{\delta}{t}\right)}{\gamma^2} \left[\frac{5}{4} - \frac{2}{(1+\lambda^2)^2} + \frac{1}{(1+4\lambda^2)} - \frac{4}{(4+\lambda^2)^2} \right] + \frac{1}{(1+\lambda^2)^2} \right.$$

$$- \frac{1}{\lambda^2}\left[\frac{\pi^2}{8} \frac{\left(\frac{\delta}{t}\right)}{\gamma^2} \left\{ 1 - 2\frac{\lambda^4}{(1+\lambda^2)^2} + \frac{\lambda^4}{(4+\lambda^2)^2} \right\} - \left\{ 1 - \frac{\lambda^4}{(1+\lambda^2)^2} \right\} \right] \frac{\left(\cosh\pi\lambda - \frac{\sinh\pi\lambda}{\pi\lambda}\right)}{1 + \frac{\sinh 2\pi\lambda}{2\pi\lambda}} +$$

$$\left. + \frac{1}{\lambda^2} \frac{\pi^2}{32} \frac{\left(\frac{\delta}{t}\right)}{\gamma^2} \left\{ 1 - \frac{16\lambda^4}{(1+4\lambda^2)^2} \right\} \frac{\cosh 2\pi\lambda - \frac{\sinh 2\pi\lambda}{2\pi\lambda}}{1 + \frac{\sinh 4\pi\lambda}{4\pi\lambda}} \right\}$$

Clamped, [illegible] at edges

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$$F = E\left(\frac{a}{R}\right)^2 \frac{f}{8} \frac{1}{\left(\frac{\pi}{a}\right)^2}\left[\left(1-\frac{f\pi^2}{16}\right)\frac{1}{\lambda^2}\cos\frac{\pi y}{b} + \left(1-\frac{f\pi^2}{8}\right)\frac{\lambda^2}{(1+\lambda^2)^2}\cos\frac{\pi x}{a}\cos\frac{\pi y}{b}\right.$$

$$- \frac{f\pi^2}{16}\left\{\lambda^2\cos\frac{\pi x}{a} + \frac{\lambda^2}{16}\cos\frac{2\pi x}{a} + \frac{1}{16\lambda^2}\cos\frac{2\pi y}{b} + \frac{\lambda^2}{(1+4\lambda^2)^2}\cos\frac{\pi x}{a}\cos\frac{2\pi y}{b} + \frac{\lambda^2}{(4+\lambda^2)^2}\cos\frac{2\pi x}{a}\cos\frac{\pi y}{b}\right\}$$

$$\left. + a_0\left(\frac{\pi x}{a}\right)^2 + b_0\left(\frac{\pi y}{b}\right)^2\right]$$

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} = E\left(\frac{a}{R}\right)^2 \frac{f}{8}\left[\left(\frac{f\pi^2}{16}-1\right)\cos\frac{\pi y}{b} + \left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^4}{(1+\lambda^2)^2}\cos\frac{\pi x}{a}\cos\frac{\pi y}{b}\right.$$

$$\left. + \frac{f\pi^2}{16}\left\{\frac{1}{4}\cos\frac{2\pi y}{b} + \frac{4\lambda^4}{(1+4\lambda^2)^2}\cos\frac{\pi x}{a}\cos\frac{2\pi y}{b} + \frac{\lambda^4}{(4+\lambda^2)^2}\cos\frac{2\pi x}{a}\cos\frac{\pi y}{b}\right\} + 2b_0\lambda^2\right]$$

$$\frac{1}{2}\left[\left(\frac{\partial w_1}{\partial x}\right)^2 - \left(\frac{\partial w_2}{\partial x}\right)^2\right] = \left(\frac{a}{R}\right)^2 \frac{f}{8}\left[-\left(\frac{\pi x}{a}\right)\sin\frac{\pi x}{a}\left(1+\cos\frac{\pi y}{b}\right) + \frac{3f\pi^2}{64} + \frac{f\pi^2}{16}\cos\frac{\pi y}{b} + \frac{f\pi^2}{64}\cos\frac{2\pi y}{b}\right.$$

$$\left. - \frac{3f\pi^2}{64}\cos\frac{2\pi x}{a} - \frac{f\pi^2}{16}\cos\frac{2\pi x}{a}\cos\frac{\pi y}{b} - \frac{f\pi^2}{64}\cos\frac{2\pi x}{a}\cos\frac{2\pi y}{b}\right]$$

$$\frac{\partial u}{\partial x} = \left(\frac{a}{R}\right)^2 \frac{f}{g}\Big[+ \left(\frac{\pi x}{a}\right)\sin\frac{\pi x}{a} - \frac{3}{64}(f\pi^2) + \frac{3}{64}(f\pi^2)\cos\frac{2\pi x}{a} + 2b_0\lambda^2$$

$$+ \left\{ \left(\frac{\pi x}{a}\right)\sin\frac{\pi x}{a} - 1 + \left(\frac{f\pi^2}{g} - 1\right)\frac{\lambda^4}{(1+\lambda^2)^2}\cos\frac{\pi x}{a} + \frac{f\pi^2}{16}\left(\frac{\lambda^4}{(4+\lambda^2)^2} + 1\right)\cos\frac{2\pi x}{a} \right\}\cos\frac{\pi y}{b} +$$

$$+ \frac{f\pi^2}{16}\left\{ \frac{4\lambda^4}{(1+4\lambda^2)^2}\cos\frac{\pi x}{a} + \frac{1}{4}\cos\frac{2\pi x}{a} \right\}\cos\frac{2\pi y}{b} \Big]$$

$u = 0$ at $x = a$, . gives $\quad 1 - \frac{3}{64}f\pi^2 + 2b_0\lambda^2 = 0$

$$\therefore \quad \boxed{2b_0\lambda^2 = \left(\frac{3}{64}f\pi^2 - 1\right)}$$

$$\sigma_y = \frac{\partial^2 F}{\partial x^2} = E\left(\frac{a}{R}\right)^2 \frac{f}{g}\Big[\left(\frac{f\pi^2}{g} - 1\right)\frac{\lambda^2}{(1+\lambda^2)^2}\cos\frac{\pi x}{a}\cos\frac{\pi y}{b} + \frac{f\pi^2}{16}\Big\{ \lambda^2\cos\frac{\pi x}{a} + \frac{\lambda^2}{4}\cos\frac{2\pi x}{a}$$

$$+ \frac{\lambda^2}{(1+4\lambda^2)^2}\cos\frac{\pi x}{a}\cos\frac{2\pi y}{b} + \frac{4\lambda^2}{(4+\lambda^2)^2}\cos\frac{2\pi x}{a}\cos\frac{\pi y}{b} \Big\} + 2a_0 \Big]$$

$$\boxed{-\frac{\sigma}{E} = \left(\frac{a}{R}\right)^2 \frac{f}{g}\,(2a_0)}$$

$$\left(\frac{v}{b}\right)_{y=b} = -\left(\frac{a}{R}\right)^2 \frac{f\lambda^2}{8}\,\frac{3}{64}(f\pi^2) - \frac{\sigma}{E}$$

$$\boxed{\frac{\Delta p}{Eabt} = -\left(\frac{\sigma}{E}\right)^2 - \left(\frac{a}{R}\right)^2\left(\frac{f}{8}\right)^2\left(\frac{3}{8}\pi^2\lambda^2\right)\frac{\sigma}{E}}$$

$$\frac{\sigma_x}{E} = \left(\frac{a}{R}\right)^2 \frac{f}{8}\Big[\left(\frac{3}{64}f\pi^2 - 1\right) + \left\{\left(\frac{f\pi^2}{16}-1\right) + \left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^4}{(1+\lambda^2)^2}\cos\frac{\pi x}{a} + \frac{f\pi^2}{16}\frac{\lambda^4}{(4+\lambda^2)^2}\cos\frac{2\pi x}{a}\right\}\cos\frac{\pi y}{b}$$
$$+ \frac{f\pi^2}{16}\left\{\frac{1}{4} + \frac{4\lambda^4}{(1+4\lambda^2)^2}\cos\frac{\pi x}{a}\right\}\cos\frac{2\pi y}{b}\Big]$$

$$\frac{1}{2ab}\int_0^a\int_0^b\left(\frac{\sigma_x}{E}\right)^2 dx\,dy = \left(\frac{a}{R}\right)^4\left(\frac{f}{8}\right)^2\Big[\frac{1}{2}\left(\frac{3}{64}f\pi^2 - 1\right)^2 + \frac{1}{4}\left(\frac{f\pi^2}{16}-1\right)^2 + \frac{1}{8}\left(\frac{f\pi^2}{8}-1\right)^2\frac{\lambda^8}{(1+\lambda^2)^4}$$
$$+ \frac{1}{8}\left(\frac{f\pi^2}{16}\right)^2\frac{\lambda^8}{(4+\lambda^2)^4} + \left(\frac{f\pi^2}{16}\right)^2\left\{\frac{1}{4}\,\frac{1}{16} + \frac{1}{8}\,\frac{16\lambda^8}{(1+4\lambda^2)^4}\right\}\Big]$$

$$\frac{\sigma_y}{E} = -\frac{\sigma}{E} + \left(\frac{a}{R}\right)^2\frac{f}{8}\Big[\frac{f\pi^2}{16}\lambda^2\cos\frac{\pi x}{a} + \frac{f\pi^2}{64}\lambda^2\cos\frac{2\pi x}{a}$$
$$+ \left\{\left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^2}{(1+\lambda^2)^2}\cos\frac{\pi x}{a} + \frac{f\pi^2}{16}\frac{4\lambda^2}{(4+\lambda^2)^2}\cos\frac{2\pi x}{a}\right\}\cos\frac{\pi y}{b}$$
$$+ \frac{f\pi^2}{16}\frac{\lambda^2}{(1+4\lambda^2)^2}\cos\frac{\pi x}{a}\cos\frac{2\pi y}{b}\Big]$$

$$\frac{1}{2ab}\int_0^a\int_0^b\left(\frac{\sigma_y}{E}\right)^2 dx\,dy = \frac{1}{2}\left(\frac{\sigma}{E}\right)^2 + \left(\frac{a}{R}\right)^4\left(\frac{f}{8}\right)^2\Bigg[\frac{1}{4}\left(\frac{f\pi^2}{16}\lambda^2\right)^2 + \frac{1}{4}\left(\frac{f\pi^2}{64}\lambda^2\right)^2$$

$$+\frac{1}{8}\left(\frac{f\pi^2}{8}-1\right)^2\frac{\lambda^4}{(1+\lambda^2)^4} + \frac{1}{8}\left(\frac{f\pi^2}{16}\right)^2\frac{16\lambda^4}{(4+\lambda^2)^4} + \frac{1}{8}\left(\frac{f\pi^2}{16}\right)^2\frac{\lambda^4}{(1+4\lambda^2)^4}\Bigg]$$

$$\frac{\tau_{xy}}{E} = \left(\frac{a}{R}\right)^2\frac{f}{8}\Bigg[\left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^3}{(1+\lambda^2)^2}\cos\frac{\pi x}{a}\cos\frac{\pi y}{b} + \frac{f\pi^2}{16}\frac{2\lambda^3}{(1+4\lambda^2)^2}\cos\frac{\pi x}{a}\cos\frac{2\pi y}{b} + \frac{f\pi^2}{16}\frac{2\lambda^3}{(4+\lambda^2)^2}\cos\frac{2\pi x}{a}\cos\frac{\pi y}{b}\Bigg]$$

$$= \left(\frac{a}{R}\right)^2\frac{f}{8}\Bigg[\left\{\left(\frac{f\pi^2}{8}-1\right)\frac{\lambda^3}{(1+\lambda^2)^2}\cos\frac{\pi x}{a} + \frac{f\pi^2}{16}\frac{2\lambda^3}{(4+\lambda^2)^2}\cos\frac{2\pi x}{a}\right\}\cos\frac{\pi y}{b}$$

$$+ \frac{f\pi^2}{16}\frac{2\lambda^3}{(1+4\lambda^2)^2}\cos\frac{\pi x}{a}\cos\frac{2\pi y}{b}\Bigg]$$

$$\frac{1}{ab}\int_0^a\int_0^b\left(\frac{\tau_{xy}}{E}\right)^2 dx\,dy = \left(\frac{a}{R}\right)^4\left(\frac{f}{8}\right)^2\Bigg[\frac{1}{4}\left(\frac{f\pi^2}{8}-1\right)^2\frac{\lambda^6}{(1+\lambda^2)^2} + \frac{1}{4}\left(\frac{f\pi^2}{16}\right)^2\frac{4\lambda^6}{(4+\lambda^2)^2} + \frac{1}{4}\left(\frac{f\pi^2}{16}\right)^2\frac{4\lambda^6}{(1+4\lambda^2)^4}\Bigg]$$

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$$\frac{1}{2}\left(\frac{3}{64}f\pi^2-1\right)^2+\frac{1}{4}\left(\frac{f\pi^2}{16}-1\right)^2+\frac{1}{4}\left(\frac{f\pi^2}{16}\lambda^2\right)^2+\frac{1}{4}\left(\frac{f\pi^2}{64}\lambda^2\right)^2+\frac{1}{8}\left(\frac{f\pi^2}{8}-1\right)^2\frac{\lambda^4}{(1+\lambda^2)^2}$$

$$+\frac{1}{8}\left(\frac{f\pi^2}{16}\right)^2\frac{\lambda^4}{(4+\lambda^2)^2}+\left(\frac{f\pi^2}{16}\right)^2\frac{1}{64}+\frac{1}{8}\left(\frac{f\pi^2}{16}\right)^2\frac{\lambda^4}{(1+4\lambda^2)^2}$$

$$H_1(\lambda)=\frac{1}{2}\left(\frac{3}{64}\right)^2+\frac{1}{4}\left(\frac{1}{16}\right)^2+\frac{1}{4}\left(\frac{\lambda^2}{16}\right)^2+\frac{1}{4}\left(\frac{\lambda^2}{64}\right)^2+\frac{1}{8}\frac{1}{64}\frac{\lambda^4}{(1+\lambda^2)^2}+\frac{1}{8}\frac{1}{256}\frac{\lambda^4}{(4+\lambda^2)^2}$$

$$+\frac{1}{8}\frac{1}{256}\frac{\lambda^4}{(1+4\lambda^2)^2}+\frac{1}{64}\frac{1}{256}$$

$$\boxed{H_1(\lambda)=\frac{35}{16384}+\frac{17}{16384}\lambda^4+\frac{1}{512}\frac{\lambda^4}{(1+\lambda^2)^2}+\frac{1}{2048}\frac{\lambda^4}{(4+\lambda^2)^2}+\frac{1}{2048}\frac{\lambda^4}{(1+4\lambda^2)^2}}$$

$$H_2(\lambda)=\frac{3}{64}+\frac{1}{32}+\frac{1}{32}\frac{\lambda^4}{(1+\lambda^2)^2}\quad,\quad\boxed{H_2(\lambda)=\frac{5}{64}+\frac{1}{32}\frac{\lambda^4}{(1+\lambda^2)^2}}$$

$$\boxed{H_3(\lambda)=\frac{3}{4}+\frac{f}{8}\frac{\lambda^4}{(1+\lambda^2)^2}}$$

Y the bending energy $= \frac{1}{24}\left(\frac{t}{R}\right)^2\left(\frac{f}{8}\right)^2\pi^4\left\{\frac{3}{4}(1+\lambda^4)+\frac{1}{2}\lambda^2\right\}$

$$= \left(\frac{t}{R}\right)^2\left(\frac{f}{8}\right)^2\pi^4\left\{\frac{1}{32}(1+\lambda^4)+\frac{\lambda^2}{48}\right\}$$

The total potential of the system

$$= \left(\frac{a}{R}\right)^4\left(\frac{f}{8}\right)^2\left[H_1(f\pi^2)^2 - H_2(f\pi^2) + H_3\right] + \left(\frac{t}{R}\right)^2\left(\frac{f}{8}\right)^2\pi^4\left\{\frac{1+\lambda^4}{32}+\frac{\lambda^2}{48}\right\}$$

$$-\frac{1}{2}\left(\frac{\sigma}{E}\right)^2 - \left(\frac{a}{R}\right)^2\left(\frac{f}{8}\right)^2\frac{\sigma}{E}\left(\frac{3}{8}\pi^2\lambda^2\right)$$

Thus

$$\frac{3}{8}\frac{\sigma}{E}\pi^2\lambda^2 = \left(\frac{a}{R}\right)^2\left[2H_1(f\pi^2)^2 - \frac{3}{2}H_2(f\pi^2) + H_3\right]$$

$$+ \left(\frac{t}{R}\right)^2\pi^4\left\{\frac{1+\lambda^4}{32}+\frac{\lambda^2}{48}\right\}\frac{1}{\left(\frac{a}{R}\right)^2}$$

$$\lambda^2 K = \gamma^2\left[\frac{16}{3}H_1 f^2\pi^2 - 4H_2 f + \frac{8}{3}H_3\frac{1}{\pi^2}\right] + \frac{\pi^2}{\gamma^2}\left[\frac{1+\lambda^4}{12}+\frac{\lambda^2}{18}\right]$$

$$= \frac{\pi^2}{\gamma^2}\left[\frac{64}{3}H_1\left(\frac{\delta}{t}\right)^2 + \left(\frac{1+\lambda^4}{12}+\frac{\lambda^2}{18}\right)\right] + \frac{\gamma^2}{\pi^2}\frac{8}{3}H_3 - 8H_2\left(\frac{\delta}{t}\right)$$

$$\boxed{\lambda^2 K = 2\left[\frac{512}{9}H_1H_3\left(\frac{\delta}{t}\right)^2 + H_3\left\{\frac{2}{9}(1+\lambda^4)+\frac{4\lambda^2}{27}\right\}\right]^{\frac{1}{2}} - 8H_2\left(\frac{\delta}{t}\right)}$$

$$\boxed{\gamma^2 = \pi^2\left[\frac{8H_1}{H_3}\left(\frac{\delta}{t}\right)^2 + \frac{1}{H_3}\left\{\frac{1}{32}(1+\lambda^2)+\frac{\lambda^2}{48}\right\}\right]}$$

thus

$$\varepsilon = -\left(\frac{\dot{a}}{R}\right)^2 \frac{f\lambda^2}{8}\frac{3}{64}\left(f\pi^2\right) - \frac{\sigma}{E}$$

$$\frac{\varepsilon}{\left(\frac{t}{R}\right)} = -K - \frac{3}{128}\pi^2\lambda^2\frac{\left(\frac{\delta}{t}\right)^2}{\gamma^2}$$

$$\boxed{\left(\frac{\varepsilon}{\frac{t}{R}}\right) = -K - \frac{3}{128}\lambda^2\left(\frac{\delta}{t}\right)^2\frac{1}{\sqrt{\frac{8H_1}{H_3}\left(\frac{\delta}{t}\right)^2 + \frac{1}{H_3}\left(\frac{1+\lambda^2}{32} + \frac{\lambda^2}{48}\right)}}}$$

$$K = 2\left\{A\left(\frac{\delta}{t}\right)^2 + B\right\}^{\frac{1}{2}} - C\left(\frac{\delta}{t}\right)$$

$$2A\left(\frac{\delta}{t}\right) = C\left\{A\left(\frac{\delta}{t}\right)^2 + B\right\}^{\frac{1}{2}}$$

$$\left(4A^2 - C^2A\right)\left(\frac{\delta}{t}\right)^2 = C^2B \qquad \left(\frac{\delta}{t}\right)^2 = \frac{C^2B}{4A^2 - C^2A} = \frac{B}{\left(\frac{2A}{C}\right)^2 - A}$$

$$H_1 = \frac{1}{2048}\left\{\frac{35}{8} + \frac{17}{8}\lambda^4 + 4\frac{\lambda^4}{(1+\lambda^2)^2} + \frac{\lambda^4}{(4+\lambda^2)^2} + \frac{\lambda^4}{(1+4\lambda^2)^2}\right\}, \quad H_2 = \frac{1}{32}\left[2.5 + \frac{\lambda^4}{(1+\lambda^2)^2}\right]$$

λ	λ^4	$\frac{\lambda^4}{(1+\lambda^2)^2}$	$\frac{\lambda^4}{(4+\lambda^2)^2}$	$\frac{\lambda^4}{(1+4\lambda^2)^2}$	$2048\ H_1$	H_1/λ^2	$32\ H_2$	H_2/λ^2
0.05	0.00000625	0.00000622	0.00000039	0.00000613	4.375044	0.854501	2.500006	31.2500
0.10	0.0001	0.00009803	0.00000622	0.00009246	4.375703	0.213657	2.500098	7.812806
0.15	0.00050625	0.00048422	0.00003129	0.00042610	4.378470	0.0950189	2.500484	3.472894
0.20	0.0016	0.00147929	0.00009803	0.00118906	4.385604	0.0535352	2.501479	1.954280
0.30	0.0081	0.00681761	0.00048422	0.00437933	4.424346	0.0240036	2.506818	0.8704229
0.40	0.0256	0.01902497	0.00147929	0.00951814	4.516497	0.0137833	2.519025	0.4919971
0.50	0.0625	0.0400000	0.00346021	0.01562500	4.686898	0.00915410	2.54000	0.3125000
0.60	0.1296	0.07006920	0.00681761	0.02176834	4.959263	0.00672643	2.570069	0.2230963
0.80	0.4096	0.15229030	0.01902497	0.03231915	5.905905	0.00450585	2.652290	0.1295064
1.00	1.0000	0.2500000	0.0400000	0.0400000	7.5800000	0.00370117	2.750000	0.0859375
1.20	2.0736	0.34829347	0.07006920	0.04537656	10.290020	0.00348918	2.848293	0.0618119
1.40	3.8416	0.43845873	0.10814828	0.04915952	14.449543	0.00359971	2.938459	0.0468504
1.60	6.5536	0.51710643	0.15229030	0.05187371	20.573990	0.00392418	3.017106	0.0368299
1.80	10.4976	0.58392667	0.20026861	0.05386655	29.272242	0.00441145	3.083927	0.0297447
2.00	16.000	0.6400000	0.2500000	0.05536332	41.240363	0.00503422	3.140000	0.0245313
2.20	23.4256	0.68217452	0.29976864	0.05651284	57.239378	0.00577457	3.182175	0.0205461

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$$H_3 = \frac{1}{8}\left(6 + \frac{\lambda^4}{(1+\lambda^4)^2}\right)$$

λ	H_3/λ^2	$\frac{\frac{2}{9}(1+\lambda^4)+\frac{4}{27}\lambda^2}{\lambda^2}$	A	B	C	K_0	$\left(\frac{9A}{C}\right)^2-A$	$(\delta/t)^2_{min}$:
0.05	300.000	89.0375	14583.5	26711.25	250.0000		−972.31	
0.10	75.0001	22.3726	911.603	1677.95	62.5024		−60.702	
0.15	33.3360	10.0297	180.198	334.350	27.7832		−11.933	
0.20	18.7546	5.71259	57.1182	107.137	15.6342		−3.7284	
0.30	8.34280	2.63778	11.3924	22.0023	6.96338		−0.68583	
0.40	4.70236	1.57259	3.68720	7.39488	3.93578		−0.17686	
0.50	3.02000	1.09259	1.57272	3.29962	2.54000		−0.03918	
0.60	2.10766	0.845432	0.806515	1.78188	1.78477		+0.010294	173.099
0.80	1.201619	0.637593	0.308014	0.766144	1.03605		+0.045527	168.283
1.00	0.781250	0.592593	0.164496	0.462963	0.687500	1.3608	+0.064498	71.7794
1.20	0.551067	0.622469	0.109384	0.343022	0.474495	1.1716	+0.086340	3.97292
1.40	0.410616	0.697082	0.0840874	0.286233	0.374803	1.0696	+0.117246	2.44130
1.60	0.318218	0.803843	0.0710397	0.255797	0.294639	1.0116	+0.161491	1.58397
1.80	0.254010	0.936735	0.0637470	0.237940	0.237958	0.9760	+0.223317	1.06548
2.00	0.207500	1.092593	0.0594262	0.226713	0.196250	0.9522	+0.307346	0.737647
2.20	0.172592	1.269617	0.0566980	0.219126	0.164369	0.9360	+0.419246	0.522667

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λ	$(\delta/t)_{min}$	$K_{min.}$	λ	$\left(\frac{\delta}{t}\right)^*$	K		$\frac{8H_1}{H_3}=D$	E	$D\left(\frac{\delta}{E}\right)^2+E$
0.05									
0.10									
0.15									
0.20									
0.30			0.3	4.82	0.303	0.0645	0.023017	0.04445	0.5792
0.40			0.4	4.53	0.400	0.0825	0.023449	0.04703	0.5282
0.50			0.5	4.13	0.486	0.1463	0.024249	0.05088	0.4645
0.60	13.1567	0.2997	0.6	3.48	0.583	0.1685	0.025531	0.05641	0.3656
0.80	4.1022	0.628[illegible]	0.8	2.55	0.690	0.1875	0.030000	0.07461	0.2697
1.00	2.6792	0.7222	1.0	1.88	0.756	0.1685	0.037900	0.10667	0.2407
1.20	1.9932	0.7780	1.2	1.30	0.816	0.1152	0.050653	0.15885	0.2445
1.40	1.5625	0.8166	1.4	1.00	0.841	0.0827	0.070133	0.23873	0.3088
1.60	1.2585	0.8429	1.6						
1.80	1.0322	0.8605	1.8						
2.00	0.85886	0.8717	2.0						
2.20	0.72296	0.8787	220		0.925				

(E) Shortening due to buckling !!

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$\lambda = 2.20$

at $\left(\frac{d}{t}\right) = 0.5$, $K = 2\left\{0.0566980 \times 0.25 + 0.219126\right\}^{\frac{1}{2}} - 0.5 \times 0.164369$

$= 0.8838$

$\left(\frac{d}{t}\right) = 1.0$ $K = 0.8860$

$\lambda = 2.00$ at $\left(\frac{d}{t}\right) = 0.5$ $K = 2\sqrt{0.241570} - 0.5 \times 0.196250 = 0.8849$

$\left(\frac{d}{t}\right) = 1.0$ $K = 2\sqrt{0.286139} - 0.196250 = 0.8736$

$\lambda = 1.8$ $\left(\frac{d}{t}\right) = 0.5$ $K = 2\sqrt{0.253877} - 0.5 \times 0.237958 = 0.8887$

$\left(\frac{d}{t}\right) = 2.0$ $K = 2\sqrt{0.492946} - 2 \times 0.237958 = 0.9283$

$\lambda = 1.6$ $\left(\frac{d}{t}\right) = 1.0$ $K = 2\sqrt{0.326837} - 0.294639 = 0.84876$

$\left(\frac{d}{t}\right) = 2.0$ $K = 2\sqrt{0.539956} - 2 \times 0.294639 = 0.8804$

$\lambda = 1.4$, $\left(\frac{d}{t}\right) = 1.0$ $K = 2\sqrt{0.370320} - 0.374803 = 0.8423$

$\left(\frac{d}{t}\right) = 2.0$ $K = 2\sqrt{0.622583} - 2 \times 0.374803 = 0.8285$

$\lambda = 1.2$. $\left(\frac{d}{t}\right) = 1.0$ $K = 2\sqrt{.452406} - 0.494495 = 0.8507$

$\left(\frac{d}{t}\right) = 3.0$ $K = 2\sqrt{1.327478} - 3 \times 0.494495 = 0.8208$

$\lambda = 1.0$ $\quad \left(\frac{\delta}{t}\right) = 1.00 \quad K = 2\sqrt{0.627459} - 0.687500 = 0.8967$

$\left(\frac{\delta}{t}\right) = 3.00 \quad K = 2\sqrt{1.943427} - 3 \times 0.687500 = 0.7256$

$\lambda = 0.8$ $\quad \left(\frac{\delta}{t}\right) = 1.00 \quad K = 2\sqrt{1.074158} - 1.03605 = 1.0368$

$\left(\frac{\delta}{t}\right) = 3.00 \quad K = 2\sqrt{3.538270} - 3 \times 1.03605 = 0.6539$

$\left(\frac{\delta}{t}\right) = 5.00 \quad K = 2\sqrt{8.466494} - 5 \times 1.03605 = 0.6392$

$\lambda = 0.6$ $\quad \left(\frac{\delta}{t}\right) = 3.00 \quad K = 2\sqrt{9.04052} - 3 \times 1.78477 = 0.6592$

$\left(\frac{\delta}{t}\right) = 5.00 \quad K = 2\sqrt{21.94476} - 5 \times 1.78477 = 0.4452$

$\left(\frac{\delta}{t}\right) = 7.00 \quad K = 2\sqrt{41.30112} - 7 \times 1.78477 = 0.3598$

$\left(\frac{\delta}{t}\right) = 10.00 \quad K = 2\sqrt{82.43338} - 10 \times 1.78477 = 0.3109$

$\left(\frac{\delta}{t}\right) = 15.00 \quad K = 2\sqrt{183.24776} - 15 \times 1.78477 = 0.3023$

$\lambda = 0.5$ $\quad \left(\frac{\delta}{t}\right) = 3.00 \quad K = 2\sqrt{17.45410} - 3 \times 2.54 = 0.7356$

$\left(\frac{\delta}{t}\right) = 4.00 \quad K = 2\sqrt{28.46314} - 4 \times 2.54 = 0.5102$

$\left(\frac{\delta}{t}\right) = 5.00 \quad K = 2\sqrt{42.81762} - 5 \times 2.54 = 0.3564$

$\lambda = 0, \qquad H_1 = \frac{35}{16384}, \qquad H_2 = \frac{5}{64}, \qquad H_3 = \frac{3}{4}$

$$\lambda K = 2\left[\frac{35\times}{32\times 3\times 4}\left(\frac{\delta}{t}\right)^2 + \frac{3}{4}\times\frac{2}{9}\right]^{\frac{1}{2}} - \frac{5}{8}\left(\frac{\delta}{t}\right)$$

$$\frac{35}{32\times 12}\left(\frac{\delta}{t}\right)^2 + \frac{1}{6} = \frac{25}{256}\left(\frac{\delta}{t}\right)^2$$

$$\left(\frac{\delta}{t}\right)^2\left[\frac{25}{256} - \frac{35}{32\times 12}\right] = \frac{1}{6}, \qquad \left(\frac{\delta}{t}\right)^2 = \frac{128}{5} = 25.6$$

$$\left(\frac{\delta}{t}\right) = 5.0597$$

$\lambda = 0.4, \qquad \left(\frac{\delta}{t}\right) = 4.00 \qquad K = 2\sqrt{66.39008} - 4\times 3.93598 = 0.5521$

$\left(\frac{\delta}{t}\right) = 5.00 \qquad K = 2\sqrt{99.57488} - 5\times 3.93598 = 0.2775$

$\left(\frac{\delta}{t}\right) = 6.00 \qquad K = 2\sqrt{140.13408} - 6\times 3.93598 = 0.0598$

$\lambda = 0.3 \qquad \left(\frac{\delta}{t}\right) = 4.00 \qquad K = 2\sqrt{204.2807} - 4\times 6.96338 = 0.7319$

$\left(\frac{\delta}{t}\right) = 5.00 \qquad K = 2\sqrt{306.8123} - 5\times 6.96338 = 0.2152$

$\left(\frac{\delta}{t}\right) = 6.00 \qquad K = 2\sqrt{432.1287} - 6\times 6.96338 = -0.2049$

Clamped, zero displacement at edges

$$K=\frac{\pi^2}{\ell^2}\left[\frac{64}{3}\left(\frac{\delta}{t}\right)^2\left\{\frac{35}{16384}\frac{1}{\lambda^2}+\frac{17}{16384}\lambda^2+\frac{1}{512}\frac{\lambda^2}{(1+\lambda^2)^2}+\frac{1}{2048}\frac{\lambda^2}{(4+\lambda^2)^2}+\frac{1}{2048}\frac{\lambda^2}{(1+4\lambda^2)^2}\right\}\right.$$

$$\left.+\left\{\frac{1}{12}\frac{1}{\lambda^2}+\frac{1}{12}\lambda^2+\frac{1}{18}\right\}\right]+\frac{\ell^2}{\pi^2}\frac{8}{3}\left\{\frac{3}{4}\frac{1}{\lambda^2}+\frac{1}{8}\frac{\lambda^2}{(1+\lambda^2)^2}\right\}-8\left(\frac{\delta}{t}\right)\left\{\frac{5}{64}\frac{1}{\lambda^2}+\frac{1}{32}\frac{\lambda^2}{(1+\lambda^2)^2}\right\}$$

$$=\frac{\pi^2}{\ell^2}\left[W^2\left\{\frac{35}{768}\frac{1}{\lambda^2}+\frac{17}{768}\lambda^2+\frac{1}{24}\frac{\lambda^2}{(1+\lambda^2)^2}+\frac{1}{96}\frac{\lambda^2}{(4+\lambda^2)^2}+\frac{1}{96}\frac{\lambda^2}{(1+4\lambda^2)^2}\right\}\right.$$

$$\left.+\frac{1}{6}\left(\frac{1}{2}\frac{1}{\lambda^2}+\frac{\lambda^2}{2}+\frac{1}{3}\right)\right]+\frac{\ell^2}{\pi^2}\left\{\frac{2}{\lambda^2}+\frac{1}{3}\frac{\lambda^2}{(1+\lambda^2)^2}\right\}-W\left\{\frac{5}{8}\frac{1}{\lambda^2}+\frac{1}{4}\frac{\lambda^2}{(1+\lambda^2)^2}\right\}$$

$$\frac{\partial K}{\partial\lambda^2}=0\ ;\ 0=\frac{\pi^2}{\ell^2}\left[W^2\left\{-\frac{35}{768}\frac{1}{\lambda^4}+\frac{17}{768}+\frac{1}{24}\frac{1}{(1+\lambda^2)^2}-\frac{1}{48}\frac{\lambda^2}{(1+\lambda^2)^3}+\frac{1}{96}\frac{1}{(4+\lambda^2)^2}-\frac{1}{192}\frac{\lambda^2}{(4+\lambda^2)^3}\right.\right.$$

$$\left.+\frac{1}{96}\frac{1}{(1+4\lambda^2)^2}-\frac{1}{192}\frac{4\lambda^2}{(1+4\lambda^2)^3}\right\}$$

$$\left.+\frac{1}{6}\left(-\frac{1}{2}\frac{1}{\lambda^4}+\frac{1}{2}\right)\right]+\frac{\ell^2}{\pi^2}\left\{-\frac{2}{\lambda^4}+\frac{1}{3}\frac{1}{(1+\lambda^2)^2}-\frac{1}{6}\frac{\lambda^2}{(1+\lambda^2)^3}\right\}-W\left\{-\frac{5}{8}\frac{1}{\lambda^4}+\frac{1}{4}\frac{1}{(1+\lambda^2)^2}\right.$$

$$\left.-\frac{1}{8}\frac{\lambda^2}{(1+\lambda^2)^3}\right\}$$

$$0=\frac{\pi^2}{\gamma^2}\left[W\left\{-\frac{35}{768}\frac{1}{\lambda^4}+\frac{17}{768}+\frac{1}{48}\frac{2+\lambda^2}{(1+\lambda^2)^3}+\frac{1}{192}\frac{8+\lambda^2}{(4+\lambda^2)^3}+\frac{1}{192}\frac{2+4\lambda^2}{(1+4\lambda^2)^3}\right\}\right.$$

$$\left.+\frac{1}{12}\left(1-\frac{1}{\lambda^4}\right)\right]+\frac{\gamma^2}{\pi^2}\left\{-\frac{2}{\lambda^4}+\frac{1}{6}\frac{2+\lambda^2}{(1+\lambda^2)^3}\right\}-W\left\{-\frac{5}{8}\frac{1}{\lambda^4}+\frac{1}{8}\frac{2+\lambda^2}{(1+\lambda^2)^3}\right\}$$

$$0=\frac{\pi^2}{\gamma^2}\left(\frac{17}{768}W+\frac{1}{12}\right)-\left[\frac{\pi^2}{\gamma^2}\left(\frac{35}{768}W+\frac{1}{12}\right)+2\frac{\gamma^2}{\pi^2}-\frac{5}{8}W\right]\frac{1}{\lambda^4}$$

$$+\left[\frac{\pi^2}{\gamma^2}\frac{1}{48}W+\frac{1}{6}\frac{\gamma^2}{\pi^2}-\frac{1}{8}W\right]\frac{2+\lambda^2}{(1+\lambda^2)^3}+\left(\frac{\pi^2}{\gamma^2}\frac{1}{192}W\right)\frac{8+\lambda^2}{(4+\lambda^2)^3}+\left(\frac{\pi^2}{\gamma^2}\frac{1}{192}W\right)\frac{2+4\lambda^2}{(1+4\lambda^2)^3}$$

$$0=\frac{\pi^2}{\gamma^2}\left(\frac{17}{768}W+\frac{1}{12}\right)\lambda^4(1+\lambda^2)^3(4+\lambda^2)^3(1+4\lambda^2)^3-\left[\frac{\pi^2}{\gamma^2}\left(\frac{35}{768}W+\frac{1}{12}\right)+2\frac{\gamma^2}{\pi^2}-\frac{5}{8}W\right](1+\lambda^2)^3(4+\lambda^2)^3(1+4\lambda^2)^3$$

$$+\left[\frac{\pi^2}{\gamma^2}\frac{1}{48}W+\frac{1}{6}\frac{\gamma^2}{\pi^2}-\frac{1}{8}W\right]\lambda^4(2+\lambda^2)(1+4\lambda^2)^3(4+\lambda^2)^3+\left(\frac{\pi^2}{\gamma^2}\frac{1}{192}W\right)\lambda^4(8+\lambda^2)(1+\lambda^2)^3(1+4\lambda^2)^3$$

$$+\left(\frac{\pi^2}{\gamma^2}\frac{1}{192}W\right)\lambda^4(2+4\lambda^2)(1+\lambda^2)^3(1+4\lambda^2)^3$$

A 11th order equation for λ^2 !!!

For any given value of γ^2 the

$$K = \frac{\pi^2}{\gamma^2}\left[\frac{64}{3}\frac{H_1}{\lambda^2}\left(\frac{\delta}{t}\right)^2 + \frac{3}{8}\frac{1}{\lambda^2}\left(\frac{2}{9}(1+\lambda^2) + \frac{4}{27}\lambda^2\right)\right] + \frac{\gamma^2}{\pi^2}\frac{8}{3}\frac{H_3}{\lambda^2} - \frac{8H_2}{\lambda^2}\left(\frac{\delta}{t}\right)$$

$$= \frac{\pi^2}{\gamma^2}\left[A(\lambda^2)\left(\frac{\delta}{t}\right)^2 + B(\lambda^2)\right] + \frac{\gamma^2}{\pi^2}C(\lambda^2) - D(\lambda^2)\left(\frac{\delta}{t}\right)$$

λ	A	B	C	D	B+C
0.05	18.2294	33.3891	800.000	250.000	833.389
0.10	4.55802	8.38973	200.000	62.5000	208.390
0.15	2.02707	3.76114	88.8960	27.4315	92.6571
0.20	1.14208	2.14222	50.0123	15.6342	52.1545
0.30	0.512077	0.988980	22.2475	6.96338	23.2365
0.40	0.294044	0.589721	12.5396	3.93598	13.1293
0.50	0.195287	0.409721	8.05333	2.54000	8.46305
0.60	0.143497	0.317037	5.62043	1.78477	5.93747
0.80	0.096125	0.239097	3.20432	1.03605	3.44342
1.00	0.078958	0.222222	2.08333	0.68750	2.30555
1.20	0.074436	0.233426	1.46951	0.494495	1.70294
1.40	0.076794	0.261406	1.09498	0.374803	1.35639
1.60	0.083716	0.301441	0.848581	0.294639	1.15002
1.80	0.094111	0.351275	0.677360	0.237958	1.02864
2.00	0.107397	0.409722	0.553333	0.196250	0.96305
2.20	0.123191	0.476106	0.460245	0.164369	0.93635

Take $\frac{\pi^2}{\gamma^2} = 1$ $\quad$ $K = A\left(\frac{\delta}{t}\right)^2 - D\left(\frac{\delta}{t}\right) + (B+C)$

We have

$$\sigma_x = \frac{E}{1-\nu^2}\left\{\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \nu\left[\frac{\partial v}{\partial y} - \frac{w}{R} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right]\right\} \qquad (1)$$

where $y = R\theta$

$$\sigma_y = \frac{E}{1-\nu^2}\left\{\frac{\partial v}{\partial y} - \frac{w}{R} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 + \nu\left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]\right\} \qquad (2)$$

$$\tau_{xy} = \frac{E}{2(1+\nu)}\left\{\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right\} \qquad (3)$$

(1) − (2)·ν,

$$\sigma_x - \nu\sigma_y = \frac{E}{1-\nu^2}\left\{(1-\nu^2)\left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]\right\}$$

$$\sigma_y - \nu\sigma_x = \frac{E}{1-\nu^2}(1-\nu^2)\left[\frac{\partial v}{\partial y} - \frac{w}{R} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right]$$

Therefore
$$\frac{\partial u}{\partial x} = \frac{1}{E}(\sigma_x - \nu\sigma_y) - \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \qquad (4)$$

$$\frac{\partial v}{\partial y} = \frac{1}{E}(\sigma_y - \nu\sigma_x) - \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 + \frac{w}{R} \qquad (5)$$

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{2(1+\nu)}{E}\tau_{xy} - \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \qquad (6)$$

Due to the equilibrium condition 5b

$$\sigma_x = \frac{\partial^2 F}{\partial y^2}, \quad \sigma_y = \frac{\partial^2 F}{\partial x^2}, \quad \tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y}$$

From (4), (5), (6)

$$\frac{1}{E}\left(\frac{\partial^2 \sigma_x}{\partial y^2} - \nu\frac{\partial^2 \sigma_y}{\partial y^2} + \frac{\partial^2 \sigma_y}{\partial x^2} - \nu\frac{\partial^2 \sigma_x}{\partial x^2}\right) - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \cancel{\frac{\partial w}{\partial x}\frac{\partial^3 w}{\partial x \partial y^2}}$$

$$-\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \cancel{\frac{\partial w}{\partial y}\frac{\partial^3 w}{\partial x^2 \partial y}} + \frac{1}{R}\frac{\partial^2 w}{\partial x^2}$$

$$= \frac{2(1+\nu)}{E}\frac{\partial^2 \tau_{xy}}{\partial x \partial y} - \frac{\partial}{\partial y}\left(\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x \partial y}\right) - \frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2}\frac{\partial w}{\partial y}\right)$$

$$= \frac{2(1+\nu)}{E}\frac{\partial^2 \tau_{xy}}{\partial x \partial y} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \cancel{\frac{\partial w}{\partial x}\frac{\partial^3 w}{\partial x \partial y^2}} - \cancel{\frac{\partial^3 w}{\partial x^2 \partial y}\frac{\partial w}{\partial y}} - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}$$

or

$$\frac{\partial^2 \sigma_x}{\partial y^2} - \nu\frac{\partial^2 \sigma_y}{\partial y^2} + \frac{\partial^2 \sigma_y}{\partial x^2} - \nu\frac{\partial^2 \sigma_x}{\partial x^2} - 2(1+\nu)\frac{\partial^2 \tau_{xy}}{\partial x \partial y}$$

$$= E\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \frac{1}{R}\frac{\partial^2 w}{\partial x^2}\right]$$

$$\boxed{\Delta\Delta F = E\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \frac{1}{R}\frac{\partial^2 w}{\partial x^2}\right]} \qquad (I)$$

$$\boxed{\frac{Et^2}{12(1-\nu^2)}\Delta\Delta\Delta\Delta w + \frac{E}{R^2}\frac{\partial^4 w}{\partial x^4} + \Delta\Delta\left(\frac{\partial^2 F}{\partial y^2}\frac{\partial^2 w}{\partial x^2} - 2\frac{\partial^2 F}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right) = 0} \qquad (II)$$

$$\frac{w}{R} = f_1 \cos\frac{ny}{R}\cos\frac{mx}{R} + f_2$$

$$\frac{\partial w}{\partial y} = -f_1 n \sin\frac{ny}{R}\cos\frac{mx}{R}$$

$$\frac{\partial^2 w}{\partial x^2} = -f_1\frac{m^2}{R}\cos\frac{ny}{R}\cos\frac{mx}{R}, \qquad \frac{\partial^2 w}{\partial y^2} = -f_1\frac{n^2}{R}\cos\frac{ny}{R}\cos\frac{mx}{R}.$$

$$\frac{\partial^2 w}{\partial x\partial y} = f_1\frac{nm}{R}\sin\frac{ny}{R}\sin\frac{mx}{R}.$$

$$\Delta\Delta F = E\left[f_1^2\frac{m^2n^2}{R^2}\left\{\sin^2\frac{ny}{R}\sin^2\frac{mx}{R} - \cos^2\frac{ny}{R}\cos^2\frac{mx}{R}\right\} + f_1\frac{m^2}{R^2}\cos\frac{ny}{R}\cos\frac{mx}{R}\right]$$

$$= E\left[-\frac{1}{2}f_1^2\frac{m^2n^2}{R^2}\left(\cos\frac{2ny}{R} + \cos\frac{2mx}{R}\right) + f_1\frac{m^2}{R^2}\cos\frac{ny}{R}\cos\frac{mx}{R}\right]$$

$$F = -\frac{1}{2}\sigma y^2 + \frac{1}{2}\alpha x^2 + E\left[f_1\frac{m^2}{R^2}\frac{\cos\frac{ny}{R}\cos\frac{mx}{R}}{\left\{\left(\frac{n}{R}\right)^2+\left(\frac{m}{R}\right)^2\right\}^2}\right.$$

$$\left. -\frac{1}{2}f_1^2\frac{m^2n^2}{R^2}\left\{\left(\frac{R}{2n}\right)^4\cos\frac{2ny}{R} + \left(\frac{R}{2m}\right)^4\cos\frac{2mx}{R}\right\}\right]$$

$$\sigma_x = -\sigma + E\left[-f_1\frac{m^2n^2}{(n^2+m^2)^2}\cos\frac{ny}{R}\cos\frac{mx}{R} + \frac{1}{8}f_1^2m^2\cos\frac{2ny}{R}\right]$$

$$\sigma_y = \alpha + E\left[-f_1\frac{m^4}{(n^2+m^2)^2}\cos\frac{ny}{R}\cos\frac{mx}{R} + \frac{1}{8}f_1^2n^2\cos\frac{2mx}{R}\right]$$

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$$\frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{E}(\alpha + \nu\sigma) + \frac{1}{8} f_1^2 \left\{ n^2 \cos\frac{2mx}{R} - \nu m^2 \cos\frac{2ny}{R} \right\}$$

$$- f_1 m^2 \frac{m^2 - \nu n^2}{(n^2+m^2)^2} \cos\frac{ny}{R} \cos\frac{mx}{R}$$

$$\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 = \frac{1}{8} f_1^2 n^2 \left(1 - \cos\frac{2ny}{R}\right)\left(1 + \cos\frac{2mx}{R}\right)$$

$$= \frac{1}{8} f_1^2 n^2 \left(1 + \cos\frac{2mx}{R} - \cos\frac{2ny}{R} - \cos\frac{2ny}{R}\cos\frac{2mx}{R}\right)$$

$$\therefore \quad \frac{\partial v}{\partial y} = \frac{1}{E}(\sigma_y - \nu\sigma_x) - \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 + \frac{w}{R}$$

$$\frac{\partial v}{\partial y} = \left[\frac{1}{E}(\alpha + \nu\sigma) - \frac{1}{8} f_1^2 n^2 + f_2\right] + \frac{1}{8} f_1^2 (n^2 - \nu m^2) \cos\frac{2ny}{R}$$

$$+ f_1 \left\{ 1 - \frac{m^2(m^2 - \nu n^2)}{(n^2+m^2)^2} \right\} \cos\frac{ny}{R} \cos\frac{mx}{R}$$

$$+ \frac{1}{8} f_1^2 n^2 \cos\frac{2ny}{R} \cos\frac{2mx}{R}$$

Put $\boxed{\frac{1}{E}(\alpha + \nu\sigma) - \frac{1}{8} f_1^2 n^2 + f_2 = 0}$ (III)

$$\boxed{\begin{aligned} \frac{v}{R} = {} & \frac{1}{16} f_1^2 \frac{(n^2 - \nu m^2)}{n} \sin\frac{2ny}{R} + f_1 \left\{ \frac{1}{n} - \frac{m^2(m^2 - \nu n^2)}{n(n^2+m^2)^2} \right\} \sin\frac{ny}{R} \cos\frac{mx}{R} \\ & + \frac{1}{16} f_1^2 n \sin\frac{2ny}{R} \cos\frac{2mx}{R} \end{aligned}}$$

$$\frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{E}(-\sigma - \nu\alpha) + \frac{1}{8}f_1^2\left[m^2\cos\frac{2ny}{R} - \nu n^2\cos\frac{2mx}{R}\right]$$

$$- f_1 m^2\frac{n^2 - \nu m^2}{(n^2+m^2)^2}\cos\frac{ny}{R}\cos\frac{mx}{R}$$

$$\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 = \frac{1}{8}f_1^2 m^2\left(1 + \cos\frac{2ny}{R} - \cos\frac{2mx}{R} - \cos\frac{2ny}{R}\cos\frac{2mx}{R}\right)$$

$$\frac{\partial u}{\partial x} = \left[\frac{1}{E}(-\sigma - \nu\alpha) - \frac{1}{8}f_1^2 m^2\right] + \frac{1}{8}f_1^2(m^2 - \nu n^2)\cos\frac{2mx}{R}$$

$$- f_1\frac{m^2(n^2 - \nu m^2)}{(n^2+m^2)^2}\cos\frac{ny}{R}\cos\frac{mx}{R} + \frac{1}{8}f_1^2 m^2\cos\frac{2ny}{R}\cos\frac{2mx}{R}$$

$$\boxed{\begin{aligned}\frac{u}{R} = \frac{x}{R}\left\{\frac{1}{E}(-\sigma - \nu\alpha) - \frac{1}{8}f_1^2 m^2\right\} + \frac{1}{16}f_1^2\frac{(m^2 - \nu n^2)}{m}\sin\frac{2mx}{R} \\ - f_1\frac{m(n^2 - \nu m^2)}{(n^2+m^2)^2}\cos\frac{ny}{R}\sin\frac{mx}{R} + \frac{1}{16}f_1^2 m\cos\frac{2ny}{R}\sin\frac{2mx}{R}\end{aligned}}$$

The wave length in x-direction

$$\frac{m l_x}{R} = 2\pi \qquad l_x = \frac{2\pi R}{m}$$

The increase in potential of σ in one length l_x,

$$\boxed{\wp = \sigma t\, 2\pi R \cdot \frac{2\pi R}{m}\left\{\frac{1}{E}(-\sigma - \nu\alpha) - \frac{1}{8}f_1^2 m^2\right\}}$$

the extensional energy = $\mathcal{E}_1$

$$\mathcal{E}_1 = \frac{t}{2E}\int_0^{2\pi R}\int_{-\frac{\pi R}{m}}^{+\frac{\pi R}{m}} \left[(\sigma_x+\sigma_y)^2 - 2(1+\nu)(\sigma_x\sigma_y - \tau_{xy}^2)\right] dx\,dy$$

$$\sigma_x+\sigma_y = (-\sigma+\alpha) + E\Big[- f_1 \frac{m^2}{n^2+m^2}\cos\frac{ny}{R}\cos\frac{mx}{R} + \frac{1}{8} f_1^2 m^2 \cos\frac{2ny}{R} + \frac{1}{8} f_1^2 n^2 \cos\frac{2mx}{R} \Big]$$

$$\mathcal{E}_{1_a} = \frac{t}{2E}\Big[(-\sigma+\alpha)^2\, 2\pi R\, \frac{2\pi R}{m} + E^2 \Big\{ f_1^2 \frac{m^4}{(n^2+m^2)^2}\, \pi R \frac{\pi R}{m} + \frac{1}{64} f_1^4 m^4\, 2\pi R \frac{\pi R}{m} + \frac{1}{64} f_1^2 n^4\, 2\pi R\frac{\pi R}{m}\Big\}\Big]$$

$$= \frac{t}{E}(\pi R)^2 \frac{1}{m}\Big[2(-\sigma+\alpha)^2 + E^2 \Big\{ \frac{1}{2}\frac{m^4}{(n^2+m^2)^2} f_1^2 + \frac{1}{64} f_1^4 (m^4+n^4)\Big\}\Big]$$

$$\boxed{\mathcal{E}_{1_a} = \frac{t}{E}\frac{(\pi R)^2}{m}\Big[2(-\sigma+\alpha)^2 + \frac{E^2 f_1^2}{2}\Big\{ \frac{m^4}{(n^2+m^2)^2} + \frac{f_1^2}{32}(m^4+n^4)\Big\}\Big]}$$

$$\mathcal{E}_{1_b} = \frac{t}{2E}\int_0^{2\pi R}\int_{-\frac{\pi R}{m}}^{+\frac{\pi R}{m}} \sigma_x\sigma_y\, dx\,dy$$

$$= \frac{t}{2E}\Big\{ -\sigma\alpha\, 2\pi R\,\frac{2\pi R}{m} + E^2 f_1^2 \frac{m^6 n^2}{(n^2+m^2)^4}\frac{(\pi R)^2}{m}\Big\}$$

$$\boxed{\mathcal{E}_{1_b} = \frac{t}{E}\frac{(\pi R)^2}{m}\Big\{ -2\sigma\alpha + \frac{E^2 f_1^2}{2}\frac{m^6 n^2}{(n^2+m^2)^4}\Big\}}$$

$$\tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y} = -E\left[f_1 \frac{m^3 n}{(n^2+m^2)^2} \sin\frac{ny}{R} \sin\frac{mx}{R} \right]$$

$$\mathcal{E}_{1c} \doteq \frac{t}{2E} \int_0^{2\pi R} \int_{-\frac{\pi R}{m}}^{+\frac{\pi R}{m}} E^2 f_1^2 \frac{m^6 n^2}{(n^2+m^2)^4} \sin^2\frac{ny}{R} \sin^2\frac{mx}{R}\, dx\, dy$$

$$\boxed{\mathcal{E}_{1c} = \frac{t}{2E} E^2 f_1^2 \frac{m^6 n^2}{(n^2+m^2)^4} \frac{(\pi R)^2}{m}}$$

$$\mathcal{E}_1 = \frac{t}{E}\frac{(\pi R)^2}{m}\left[2(-\sigma+\alpha)^2 + \frac{E^2 f_1^2}{2}\left\{ \frac{m^4}{(n^2+m^2)^2} + \frac{f_1^2}{32}(m^4+n^4) \right\} \right.$$

$$\left. -2(1+\nu)\left\{ -2\sigma\alpha + \frac{E^2 f_1^2}{2} \cancel{\frac{m^6 n^2}{(n^2+m^2)^4}} - \frac{E^2 f_1^2}{2} \cancel{\frac{m^6 n^2}{(n^2+m^2)^4}} \right\} \right]$$

$$\mathcal{E}_1 = \frac{t}{E}\frac{(\pi R)^2}{m}\left[2\left\{ (-\sigma+\alpha)^2 + 2(1+\nu)\sigma\alpha \right\} + \frac{E^2 f_1^2}{2}\left\{ \frac{m^4}{(n^2+m^2)^2} + \frac{f_1^2(m^4+n^4)}{32} \right\} \right]$$

$$K_1 = \frac{\partial^2 w}{\partial x^2}$$

$$K_2 = \frac{\partial^2 w}{\partial y^2} + \frac{\partial}{\partial y}\left(\frac{v}{R}\right) \qquad K_{12} = \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial y} + \frac{v}{R} \right)$$

$$\boxed{K_1 = -f_1 \frac{m^2}{R} \cos\frac{ny}{R} \cos\frac{mx}{R}}$$

$$K_2 = -f_1 \frac{n^2}{R} \cos\frac{ny}{R} \cos\frac{mx}{R} + \frac{1}{8}\frac{1}{R} f_1^2 (n^2 - \nu m^2) \cos\frac{2ny}{R}$$

$$+ f_1 \frac{1}{R}\left\{ 1 - \frac{m^2(m^2-\nu n^2)}{(n^2+m^2)^2} \right\} \cos\frac{ny}{R} \cos\frac{mx}{R} + \frac{1}{8} f_1^2 \frac{1}{R} n^2 \cos\frac{2ny}{R} \cos\frac{2mx}{R}$$

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$$\boxed{K_2 = \frac{1}{R}\left[f_1 \left\{ 1 - n^2 - \frac{m^2(m^2 - \nu n^2)}{(n^2+m^2)^2} \right\} \cos\frac{ny}{R} \cos\frac{mx}{R} + \frac{1}{8} f_1^2 (n^2 - \nu m^2) \cos\frac{2ny}{R} + \frac{1}{8} f_1^2 n^2 \cos\frac{2ny}{R} \cos\frac{2mx}{R} \right]}$$

$$K_{12} = f_1 \frac{nm}{R} \sin\frac{ny}{R} \sin\frac{mx}{R} - f_1 \frac{m}{R} \left\{ \frac{1}{n} - \frac{m^2(m^2 - \nu n^2)}{n(n^2+m^2)^2} \right\} \sin\frac{ny}{R} \sin\frac{mx}{R} - \frac{1}{8}\frac{1}{R} f_1^2 mn \sin\frac{2ny}{R} \sin\frac{2mx}{R}$$

$$\boxed{K_{12} = \frac{1}{R}\left[f_1 m \left\{ n - \frac{1}{n} - \frac{m^2(m^2 - \nu n^2)}{n(n^2+m^2)^2} \right\} \sin\frac{ny}{R} \sin\frac{mx}{R} - \frac{1}{8} f_1^2 mn \sin\frac{2ny}{R} \sin\frac{2mx}{R} \right]}$$

$$\mathcal{E}_2 = \frac{Et^3}{24(1-\nu^2)}\frac{1}{R^2}\left[\frac{(\pi R)^2}{m} f_1^2 \left\{ 1 - m^2 - n^2 - \frac{m^2(m^2 - \nu n^2)}{(n^2+m^2)^2} \right\}^2 + \frac{2(\pi R)^2}{m}\frac{1}{64} f_1^4 (n^2 - \nu m^2)^2 + \frac{(\pi R)^2}{m}\frac{1}{64} f_1^4 n^4 \right.$$

$$\left. - 2(1-\nu)\left\{ - f_1^2 \frac{(\pi R)^2}{m} m^2 \left[1 - n^2 - \frac{m^2(m^2 - \nu n^2)}{(n^2+m^2)^2} \right] - f_1^2 m^2 \left[n - \frac{1}{n} - \frac{m^2(m^2 - \nu n^2)}{n(n^2+m^2)^2} \right]^2 \frac{(\pi R)^2}{m} - \frac{1}{64} f_1^4 m^2 n^2 \frac{(\pi R)^2}{m} \right\} \right]$$

$$\mathcal{E}_2 = \frac{Et}{(1-\nu^2)}\frac{1}{12}\left(\frac{t}{R}\right)^2\frac{(\pi R)^2}{m}\left[\frac{f_1^2}{2}\left\{1-m^2-n^2-\frac{m^2(m^2-\nu n^2)}{(n^2+m^2)^2}\right\}^2\right.$$

$$+\frac{1}{64}f_1^4\left\{(n^2-\nu m^2)^2+\frac{n^4}{2}\right\}+(1-\nu)f_1^2 m^2\left\{1-n^2-\frac{m^2(m^2-\nu n^2)}{(n^2+m^2)^2}\right\}$$

$$\left.+(1-\nu)f_1^2\left(\frac{m}{n}\right)^2\left\{n^2-1-\frac{m^2(m^2-\nu n^2)}{(n^2+m^2)^2}\right\}^2+(1-\nu)\frac{f_1^4}{64}m^2n^2\right]$$

The total potential $\div\ Et\frac{(\pi R)^2}{m}$

$$=\frac{1}{E^2}2\left\{(-\sigma+\alpha)^2+2(1+\nu)\sigma\alpha\right\}+\frac{f_1^2}{2}\left\{\frac{m^4}{(n^2+m^2)^2}+f_1^2\frac{(m^4+n^4)}{32}\right\}$$

$$+\frac{1}{12(1-\nu^2)}\left(\frac{t}{R}\right)^2\left[\frac{f_1^2}{2}\left\{1-m^2-n^2-\frac{m^2(m^2-\nu n^2)}{(n^2+m^2)^2}\right\}^2+\frac{1}{64}f_1^4\left\{(n^2-\nu m^2)^2+\frac{n^4}{2}\right\}\right.$$

$$+(1-\nu)f_1^2m^2\left\{1-n^2-\frac{m^2(m^2-\nu n^2)}{(n^2+m^2)^2}\right\}+(1-\nu)f_1^2\left(\frac{m}{n}\right)^2\left\{n^2-1-\frac{m^2(m^2-\nu n^2)}{(n^2+m^2)^2}\right\}^2$$

$$\left.+(1-\nu)\frac{f_1^4}{64}m^2n^2\right]-4\frac{\sigma}{E}\left[\frac{1}{E}(\sigma+\nu\alpha)+\frac{1}{8}f_1^2m^2\right]$$

$$\frac{1}{E}(\alpha+\nu\sigma)=\frac{1}{8}f_1^2n^2-f_2$$

$$\frac{\alpha}{E}=-\nu\frac{\sigma}{E}+\frac{1}{8}f_1^2n^2-f_2$$

$$\frac{1}{E}(\sigma+\nu\alpha)=\frac{\sigma}{E}(1-\nu^2)+\nu\left(\frac{1}{8}f_1^2n^2-f_2\right)$$

$$\frac{1}{E}(-\sigma+\alpha)=-(1+\nu)\frac{\sigma}{E}+\frac{1}{8}f_1^2n^2-f_2$$

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$$\frac{1}{E^2}\sigma\alpha = -\nu\left(\frac{\sigma}{E}\right)^2 + \frac{\sigma}{E}\left(\frac{1}{8}f_1^2 n^2 - f_2\right)$$

herefore

$$2\left\{\left(\frac{-\sigma+\alpha}{E}\right)^2 + 2(1+\nu)\frac{\sigma\alpha}{E^2}\right\} - 4\frac{\sigma}{E}\left[\frac{\sigma+\nu\alpha}{E} + \frac{1}{8}f_1^2 m^2\right]$$

$$= 2\left\{(1+\nu)^2\left(\frac{\sigma}{E}\right)^2 - \cancel{2(1+\nu)\frac{\sigma}{E}\left(\frac{1}{8}f_1^2 n^2 - f_2\right)} + \left(\frac{1}{8}f_1^2 n^2 - f_2\right)^2\right.$$

$$\left. - 2(1+\nu)\nu\left(\frac{\sigma}{E}\right)^2 + \cancel{2(1+\nu)\frac{\sigma}{E}\left(\frac{1}{8}f_1^2 n^2 - f_2\right)}\right\}$$

$$- 4\frac{\sigma}{E}\left[\frac{\sigma}{E}(1-\nu^2) + \nu\left(\frac{1}{8}f_1^2 n^2 - f_2\right) + \frac{1}{8}f_1^2 m^2\right]$$

$$= -2\left(\frac{\sigma}{E}\right)^2(1-\nu^2) + 2\left(\frac{1}{8}f_1^2 n^2 - f_2\right)^2 - 4\frac{\sigma}{E}\left\{\frac{1}{8}f_1^2 m^2 + \nu\left(\frac{1}{8}f_1^2 n^2 - f_2\right)\right\}$$

$$A = -2\left(\frac{\sigma}{E}\right)^2(1-\nu^2) + 2\left(\frac{1}{8}f_1^2 n^2 - f_2\right)^2 - 4\frac{\sigma}{E}\left\{\frac{1}{8}f_1^2(m^2+\nu n^2) - \nu f_2\right\}$$

$$-\left(\frac{1}{8}f_1^2 n^2 - f_2\right) + \frac{\sigma}{E}\nu = 0$$

$$\boxed{f_2 = \frac{1}{8}f_1^2 n^2 - \frac{\sigma}{E}\nu}$$

$$A = -2\left(\frac{\sigma}{E}\right)^2(1-\nu^2) + 2\left(\frac{\sigma}{E}\right)^2\nu^2 - 4\nu^2\left(\frac{\sigma}{E}\right)^2 - \frac{1}{2}\frac{\sigma}{E}f_1^2 m^2$$

$$= -2\left(\frac{\sigma}{E}\right)^2 - \frac{1}{2}\left(\frac{\sigma}{E}\right)f_1^2 m^2.$$

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$$\frac{1}{2}\left(\frac{\sigma}{E}\right)m^2 = \frac{1}{2}\left\{\frac{m^4}{(n^2+m^2)^2} + \frac{(m^4+n^4)}{16}f_1^2\right\}$$

$$+\frac{1}{12(1-\nu^2)}\left(\frac{t}{R}\right)^2\left[\frac{1}{2}\left\{1-m^2-n^2-\frac{m^2(m^2-\nu n^2)}{(n^2+m^2)^2}\right\}^2+\frac{1}{32}\left\{(n^2-\nu m^2)^2+\frac{n^4}{2}\right\}f_1^2\right.$$

$$+(1-\nu)m^2\left\{1-n^2-\frac{m^2(m^2-\nu m^2)}{(n^2+m^2)^2}\right\}+(1-\nu)\left(\frac{m}{n}\right)^2\left\{n^2-1-\frac{m^2(m^2-\nu n^2)}{(m^2+n^2)^2}\right\}^2$$

$$\left.+(1-\nu)m^2n^2\frac{f_1^2}{32}\right]$$

No Good !!!

$$\frac{w}{R} = a_{00} + a_{01}\cos\frac{ny}{R} + a_{02}\cos\frac{2ny}{R} + a_{03}\cos\frac{3ny}{R}$$

$$+ a_{10}\cos\frac{mx}{R} + a_{11}\cos\frac{mx}{R}\cos\frac{ny}{R} + a_{12}\cos\frac{mx}{R}\cos\frac{2ny}{R} + a_{13}\cos\frac{mx}{R}\cos\frac{3ny}{R}$$

$$+ a_{20}\cos\frac{2mx}{R} + a_{21}\cos\frac{2mx}{R}\cos\frac{ny}{R} + a_{22}\cos\frac{2mx}{R}\cos\frac{2ny}{R} + a_{23}\cos\frac{2mx}{R}\cos\frac{3ny}{R}$$

$$+ a_{30}\cos\frac{3mx}{R} + a_{31}\cos\frac{3mx}{R}\cos\frac{ny}{R} + a_{32}\cos\frac{3mx}{R}\cos\frac{2ny}{R} + a_{33}\cos\frac{3mx}{R}\cos\frac{3ny}{R}$$

$$\frac{1}{R}\frac{\partial^2 w}{\partial x^2} = -\left(\frac{m}{R}\right)^2\left\{a_{10}\cos\frac{mx}{R} + a_{11}\cos\frac{mx}{R}\cos\frac{ny}{R} + a_{12}\cos\frac{mx}{R}\cos\frac{2ny}{R} + a_{13}\cos\frac{mx}{R}\cos\frac{3ny}{R}\right\}$$

$$-\left(\frac{m}{R}\right)^2\left\{4a_{20}\cos\frac{2mx}{R} + 4a_{21}\cos\frac{2mx}{R}\cos\frac{ny}{R} + 4a_{22}\cos\frac{2mx}{R}\cos\frac{2ny}{R} + 4a_{23}\cos\frac{2mx}{R}\cos\frac{3ny}{R}\right\}$$

$$-\left(\frac{m}{R}\right)^2\left\{9a_{30}\cos\frac{3mx}{R} + 9a_{31}\cos\frac{3mx}{R}\cos\frac{ny}{R} + 9a_{32}\cos\frac{3mx}{R}\cos\frac{2ny}{R} + 9a_{33}\cos\frac{3mx}{R}\cos\frac{3ny}{R}\right\}$$

$$\frac{1}{R}\frac{\partial^2 w}{\partial y^2} = -\left(\frac{n}{R}\right)^2\left\{a_{01}\cos\frac{ny}{R} + 4a_{02}\cos\frac{2ny}{R} + 9a_{03}\cos\frac{3ny}{R}\right\}$$

$$-\left(\frac{n}{R}\right)^2\left\{a_{11}\cos\frac{mx}{R}\cos\frac{ny}{R} + 4a_{12}\cos\frac{mx}{R}\cos\frac{2ny}{R} + 9a_{13}\cos\frac{mx}{R}\cos\frac{3ny}{R}\right\}$$

$$-\left(\frac{n}{R}\right)^2\left\{a_{21}\cos\frac{2mx}{R}\cos\frac{ny}{R} + 4a_{22}\cos\frac{2mx}{R}\cos\frac{2ny}{R} + 9a_{23}\cos\frac{2mx}{R}\cos\frac{3ny}{R}\right\}$$

$$-\left(\frac{n}{R}\right)^2\left\{a_{31}\cos\frac{3mx}{R}\cos\frac{ny}{R} + 4a_{32}\cos\frac{3mx}{R}\cos\frac{2ny}{R} + 9a_{33}\cos\frac{3mx}{R}\cos\frac{3ny}{R}\right\}$$

$$\frac{1}{R}\frac{\partial^2 w}{\partial x\partial y} = \frac{mn}{R^2}\left\{a_{11}\sin\frac{mx}{R}\sin\frac{ny}{R} + 2a_{12}\sin\frac{mx}{R}\sin\frac{2ny}{R} + 3a_{13}\sin\frac{mx}{R}\sin\frac{3ny}{R}\right.$$

$$+ 2a_{21}\sin\frac{2mx}{R}\sin\frac{ny}{R} + 4a_{22}\sin\frac{2mx}{R}\sin\frac{2ny}{R} + 6a_{23}\sin\frac{2mx}{R}\sin\frac{3ny}{R}$$

$$\left. + 3a_{31}\sin\frac{3mx}{R}\sin\frac{ny}{R} + 6a_{32}\sin\frac{3mx}{R}\sin\frac{2ny}{R} + 9a_{33}\sin\frac{3mx}{R}\sin\frac{3ny}{R}\right\}$$

$$\frac{\partial^2 w}{\partial x \partial y} = \frac{mn}{R}\Big[\sin\frac{mx}{R}\Big\{ a_{11}\sin\frac{ny}{R} + 2a_{12}\sin\frac{2ny}{R} + 3a_{13}\sin\frac{3ny}{R}\Big\}$$ 587

$$+ 2\sin\frac{2mx}{R}\Big\{ a_{21}\sin\frac{ny}{R} + 2a_{22}\sin\frac{2ny}{R} + 3a_{33}\sin\frac{3ny}{R}\Big\}$$

$$+ 3\sin\frac{3mx}{R}\Big\{ a_{31}\sin\frac{ny}{R} + 2a_{32}\sin\frac{2ny}{R} + 3a_{33}\sin\frac{3ny}{R}\Big\}\Big]$$

$$\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 = \frac{m^2n^2}{R^2}\Big[\frac{1}{2}\Big(1-\cos\frac{2mx}{R}\Big)\Big\{ a_{11}\sin\frac{ny}{R} + 2a_{12}\sin\frac{2ny}{R} + 3a_{13}\sin\frac{3ny}{R}\Big\}^2$$

$$+ 2\Big(1-\cos\frac{4mx}{R}\Big)\Big\{ a_{21}\sin\frac{ny}{R} + 2a_{22}\sin\frac{2ny}{R} + 3a_{23}\sin\frac{3ny}{R}\Big\}^2$$

$$+ \frac{9}{2}\Big(1-\cos\frac{6mx}{R}\Big)\Big\{ a_{31}\sin\frac{ny}{R} + 2a_{32}\sin\frac{2ny}{R} + 3a_{33}\sin\frac{3ny}{R}\Big\}^2$$

$$+ 2\Big(\cos\frac{mx}{R} - \cos\frac{3mx}{R}\Big)\Big\{ a_{11}\sin\theta + 2a_{12}\sin 2\theta + 3a_{13}\sin 3\theta\Big\}\Big\{ a_{21}\sin\theta + 2a_{22}\sin 2\theta + 3a_{23}\sin 3\theta\Big\}$$

$$+ 6\Big(\cos\frac{mx}{R} - \cos\frac{5mx}{R}\Big)\Big\{ a_{21}\sin\theta + 2a_{22}\sin 2\theta + 3a_{23}\sin 3\theta\Big\}\Big\{ a_{31}\sin\theta + 2a_{32}\sin 2\theta + 3a_{33}\sin 3\theta\Big\}$$

$$+ 3\Big(\cos\frac{2mx}{R} - \cos\frac{4mx}{R}\Big)\Big\{ a_{31}\sin\theta + 2a_{32}\sin 2\theta + 3a_{33}\sin 3\theta\Big\}\Big\{ a_{11}\sin\theta + 2a_{12}\sin 2\theta + 3a_{13}\sin 3\theta\Big\}\Big]$$

$$(a_{11}\sin\theta + 2a_{12}\sin 2\theta + 3a_{13}\sin 3\theta)^2$$

$$= a_{11}^2\sin^2\theta + 4a_{12}^2\sin^2 2\theta + 9a_{13}^2\sin^2 3\theta + 4a_{11}a_{12}\sin\theta\sin 2\theta + 12a_{12}a_{13}\sin 2\theta\sin 3\theta$$
$$+ 6a_{13}a_{11}\sin 3\theta\sin\theta$$

$$= \frac{1}{2}a_{11}^2(1-\cos 2\theta) + 2a_{12}^2(1-\cos 4\theta) + \frac{9}{2}a_{13}^2(1-\cos 6\theta)$$

$$+ 2a_{11}a_{12}(\cos\theta - \cos 3\theta) + 6a_{12}a_{13}(\cos\theta - \cos 5\theta) + 3a_{13}a_{11}(\cos 2\theta - \cos 4\theta)$$

$$(a_{11}\sin\theta + 2a_{12}\sin 2\theta + 3a_{13}\sin 3\theta)^2$$

$$= \left(\tfrac{1}{2}a_{11}^2 + 2a_{12}^2 + \tfrac{9}{2}a_{13}^2\right) + (2a_{11}a_{12} + 6a_{12}a_{13})\cos\theta + \left(3a_{13}a_{11} - \tfrac{1}{2}a_{11}^2\right)\cos 2\theta - 2a_{11}a_{12}\cos 3\theta$$

$$- (2a_{12}^2 + 3a_{13}a_{11})\cos 4\theta - 6a_{12}a_{13}\cos 5\theta - \tfrac{9}{2}a_{13}^2\cos 6\theta$$

$$\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 = \frac{m^2 n^2}{R^2}\Big[\tfrac{1}{2}(1-\cos 2\varphi)\Big\{\left(\tfrac{1}{2}a_{11}^2 + 2a_{12}^2 + \tfrac{9}{2}a_{13}^2\right) + (2a_{11}a_{12} + 6a_{12}a_{13})\cos\theta + \left(3a_{13}a_{11} - \tfrac{1}{2}a_{11}^2\right)\cos 2\theta$$

$$- 2a_{11}a_{12}\cos 3\theta - (2a_{12}^2 + 3a_{13}a_{11})\cos 4\theta - 6a_{12}a_{13}\cos 5\theta - \tfrac{9}{2}a_{13}^2\cos 6\theta\Big\}$$

$$+2(1-\cos 4\varphi)\Big\{\left(\tfrac{1}{2}a_{21}^2 + 2a_{22}^2 + \tfrac{9}{2}a_{23}^2\right) + (2a_{21}a_{22} + 6a_{22}a_{23})\cos\theta + \left(3a_{23}a_{21} - \tfrac{1}{2}a_{21}^2\right)\cos 2\theta$$

$$- 2a_{21}a_{22}\cos 3\theta - (2a_{22}^2 + 3a_{23}a_{21})\cos 4\theta - 6a_{22}a_{23}\cos 5\theta - \tfrac{9}{2}a_{23}^2\cos 6\theta\Big\}$$

$$+\tfrac{9}{2}(1-\cos 6\varphi)\Big\{\Big(\tfrac{1}{2}a$$

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$$\frac{1}{R} w = f_1 + f_2 \cos\frac{mx}{R}\cos\frac{my}{R} + f_3 \cos\frac{2mx}{R} + f_4 \cos\frac{2my}{R}$$

$$\frac{1}{R}\frac{\partial^2 w}{\partial x^2} = -\left(\frac{m}{R}\right)^2\left[f_2\cos\frac{mx}{R}\cos\frac{my}{R} + 4f_3\cos\frac{2mx}{R}\right]$$

$$\frac{1}{R}\frac{\partial^2 w}{\partial y^2} = -\left(\frac{m}{R}\right)^2\left[f_2\cos\frac{mx}{R}\cos\frac{my}{R} + 4f_4\cos\frac{2my}{R}\right]$$

$$\frac{1}{R}\frac{\partial^2 w}{\partial x\partial y} = +\left(\frac{m}{R}\right)^2\left[f_2\sin\frac{mx}{R}\sin\frac{my}{R}\right]$$

Thus $\left(\frac{1}{R}\frac{\partial^2 w}{\partial x\partial y}\right)^2 = \left(\frac{m}{R}\right)^4\left[f_2^2\sin^2\frac{mx}{R}\sin^2\frac{my}{R}\right]$

$$\left(\frac{1}{R}\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{1}{R}\frac{\partial^2 w}{\partial y^2}\right) = \left(\frac{m}{R}\right)^4\Big[f_2^2\cos^2\frac{mx}{R}\cos^2\frac{my}{R} + 2f_2f_3\left(\cos\frac{3mx}{R}+\cos\frac{mx}{R}\right)\cos\frac{my}{R}$$
$$+ 2f_2f_4\cos\frac{mx}{R}\left(\cos\frac{3my}{R}+\cos\frac{my}{R}\right) + 16f_3f_4\cos\frac{2mx}{R}\cos\frac{2my}{R}\Big]$$

$$\Delta\Delta F = \frac{m^2}{R^2}E\Big[m^2f_2^2\left\{\sin^2\frac{mx}{R}\sin^2\frac{my}{R} - \cos^2\frac{mx}{R}\cos^2\frac{my}{R}\right\}$$
$$-m^2\Big\{2f_2f_3\left(\cos\frac{3mx}{R}+\cos\frac{mx}{R}\right)\cos\frac{my}{R} + 2f_2f_4\cos\frac{mx}{R}\left(\cos\frac{3my}{R}+\cos\frac{my}{R}\right)$$
$$+16f_3f_4\cos\frac{2mx}{R}\cos\frac{2my}{R}\Big\}$$
$$+ f_2\cos\frac{mx}{R}\cos\frac{my}{R} + 4f_3\cos\frac{2mx}{R}\Big]$$

$$= \left(\frac{m}{R}\right)^2E\Big[-\frac{m^2}{2}f_2^2\cos\frac{2mx}{R} - \frac{m^2}{2}f_2^2\cos\frac{2my}{R} - 16m^2f_3f_4\cos\frac{2mx}{R}\cos\frac{2my}{R}$$
$$-\left(2m^2f_2f_3 + 2m^2f_2f_4 - f_2\right)\cos\frac{mx}{R}\cos\frac{my}{R}$$
$$-2m^2f_2f_3\cos\frac{3mx}{R}\cos\frac{my}{R} - 2m^2f_2f_4\cos\frac{mx}{R}\cos\frac{3my}{R} + 4f_3\cos\frac{2mx}{R}\Big]$$

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$$\Delta\Delta F = \left(\frac{m}{R}\right)^2 E\Big[\, f_2\left(1-2m^2f_3-2m^2f_4\right)\cos\frac{mx}{R}\cos\frac{my}{R}$$

$$+\left(4f_3-\frac{m^2}{2}f_2^2\right)\cos\frac{2mx}{R}-\frac{m^2}{2}f_2^2\cos\frac{2my}{R}-16m^2f_3f_4\cos\frac{2mx}{R}\cos\frac{2my}{R}$$

$$-2m^2f_2f_3\cos\frac{3mx}{R}\cos\frac{my}{R}-2m^2f_2f_4\cos\frac{mx}{R}\cos\frac{3my}{R}\Big]$$

$$F=\frac{E}{\left(\frac{m}{R}\right)^2}\Big[\frac{1}{4}f_2\left(1-2m^2f_3-2m^2f_4\right)\cos\frac{mx}{R}\cos\frac{my}{R}$$

$$+\frac{1}{4}\frac{1}{4}\left(4f_3-\frac{m^2}{2}f_2^2\right)\cos\frac{2mx}{R}-\frac{1}{4}\frac{m^2}{8}f_2^2\cos\frac{2my}{R}-\frac{1}{4}m^2f_3f_4\cos\frac{2mx}{R}\cos\frac{2my}{R}$$

$$-\frac{1}{50}m^2f_2f_3\cos\frac{3mx}{R}\cos\frac{my}{R}-\frac{1}{50}m^2f_2f_4\cos\frac{mx}{R}\cos\frac{3my}{R}\Big]$$

$$\sigma_x = E\Big[-\frac{1}{4}f_2\left(1-2m^2f_3-2m^2f_4\right)\cos\frac{mx}{R}\cos\frac{my}{R}$$

$$+\frac{m^2}{8}f_2^2\cos\frac{2my}{R}+m^2f_3f_4\cos\frac{2mx}{R}\cos\frac{2my}{R}+\frac{1}{50}m^2f_2f_3\cos\frac{3mx}{R}\cos\frac{my}{R}$$

$$+\frac{9}{50}m^2f_2f_4\cos\frac{mx}{R}\cos\frac{3my}{R}\Big]-\sigma$$

$$\sigma_y = E\Big[-\frac{1}{4}f_2\left(1-2m^2f_3-2m^2f_4\right)\cos\frac{mx}{R}\cos\frac{my}{R}$$

$$+\left(\frac{m^2}{8}f_2^2-f_3\right)\cos\frac{2mx}{R}+m^2f_3f_4\cos\frac{2mx}{R}\cos\frac{2my}{R}+\frac{9}{50}m^2f_2f_3\cos\frac{3mx}{R}\cos\frac{my}{R}$$

$$+\frac{1}{50}m^2f_2f_4\cos\frac{mx}{R}\cos\frac{3my}{R}\Big]+\alpha$$

$$\frac{1}{E}(\sigma_y - \nu\sigma_x) = -\frac{1}{4} f_2 (1-\nu)(1 - 2m^2 f_3 - 2m^2 f_4) \cos\frac{mx}{R}\cos\frac{my}{R}$$

$$+ \left(\frac{m^2}{8} f_2^2 \; f_3\right)\cos\frac{2mx}{R} - \nu\frac{m^2}{8} f_2^2 \cos\frac{2my}{R} + (1-\nu) m^2 f_3 f_4 \cos\frac{2mx}{R}\cos\frac{2my}{R}$$

$$+ \frac{1}{50}(9-\nu)\, m^2 f_2 f_3 \cos\frac{3mx}{R}\cos\frac{my}{R} + \frac{1}{50}(1-9\nu)\, m^2 f_2 f_4 \cos\frac{mx}{R}\cos\frac{3my}{R}$$

$$+ \frac{1}{E}(\alpha + \nu\sigma)$$

$$-\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 = -m^2\frac{1}{2}\left\{ f_2 \cos\frac{mx}{R}\sin\frac{my}{R} + 2 f_4 \sin\frac{2my}{R}\right\}^2$$

$$= -m^2\left\{\frac{1}{8} f_2^2\left(1+\cos\frac{2mx}{R}\right)\left(1-\cos\frac{2my}{R}\right) + f_2 f_4 \cos\frac{mx}{R}\left(\cos\frac{my}{R} - \cos\frac{3my}{R}\right)\right.$$

$$\left. + f_4^2\left(1-\cos\frac{4my}{R}\right)\right\}$$

$$-\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 = \left(-\frac{m^2}{8} f_2^2 - m^2 f_4^2\right) - \frac{m^2}{8} f_2^2 \cos\frac{2mx}{R} + \frac{m^2}{8} f_2^2 \cos\frac{2my}{R}$$

$$+ \frac{m^2}{8} f_2^2 \cos\frac{2mx}{R}\cos\frac{2my}{R} - m^2 f_2 f_4 \cos\frac{mx}{R}\cos\frac{my}{R} + m^2 f_2 f_4 \cos\frac{mx}{R}\cos\frac{3my}{R}$$

$$+ m^2 f_4^2 \cos\frac{4my}{R}$$

$$\frac{w}{R} = f_1 + f_2 \cos\frac{mx}{R}\cos\frac{my}{R} + f_3 \cos\frac{2mx}{R} + f_4 \cos\frac{2my}{R}$$

$$\frac{\partial v}{\partial y} = \left\{ \frac{1}{E}(\alpha + \nu\sigma) - m^2\left(\frac{1}{8}f_2^2 + f_4^2\right) + f_1 \right\} + \cdots$$

$$\therefore \quad \boxed{f_1 = m^2\left(\frac{1}{8}f_2^2 + f_4^2\right) - \frac{1}{E}(\alpha + \nu\sigma)}$$

$$\tau_{xy} = E\Big[-\frac{1}{4}f_2\left(1 - 2m^2 f_3 - 2m^2 f_4\right)\sin\frac{mx}{R}\sin\frac{my}{R}$$

$$+ m^2 f_3 f_4 \sin\frac{2mx}{R}\sin\frac{2my}{R} + \frac{3}{50}m^2 f_2 f_3 \sin\frac{3mx}{R}\sin\frac{my}{R}$$

$$+ \frac{3}{50}m^2 f_2 f_4 \sin\frac{mx}{R}\sin\frac{3my}{R}\Big]$$

$$\sigma_x + \sigma_y = E\Big[-\frac{1}{2}f_2\left(1 - 2m^2 f_3 - 2m^2 f_4\right)\cos\frac{mx}{R}\cos\frac{my}{R}$$

$$+ \left(\frac{m^2}{8}f_2^2 - f_3\right)\cos\frac{2mx}{R} + \frac{m^2}{8}f_2^2 \cos\frac{2my}{R} + 2m^2 f_3 f_4 \cos\frac{2mx}{R}\cos\frac{2my}{R}$$

$$+ \frac{1}{5}m^2 f_2 f_3 \cos\frac{3mx}{R}\cos\frac{my}{R} + \frac{1}{5}m^2 f_2 f_4 \cos\frac{mx}{R}\cos\frac{3my}{R}\Big] + (\alpha - \sigma)$$

$$\frac{1}{R}\frac{t}{2E}\iint(\sigma_x + \sigma_y)^2\,dx\,dy$$

$$= \left(\frac{t}{R}\right)\frac{E}{2}\Big[\frac{1}{4}f_2^2\left(1 - 2m^2 f_3 - 2m^2 f_4\right)^2 + 2\left(f_3 - \frac{m^2}{8}f_2^2\right)^2 + 2\left(\frac{m^2}{8}f_2^2\right)^2$$

$$+ \left(2m^2 f_3 f_4\right)^2 + \frac{1}{25}m^4 f_2^2\left(f_3^2 + f_4^2\right) + 4\left(\frac{\alpha}{E} - \frac{\sigma}{E}\right)^2\Big]$$

$$\frac{1}{R}\frac{t}{2E}\iint(\sigma_x\sigma_y)\,dx\,dy$$

$$\sim\left(\frac{t}{R}\right)\frac{E}{2}\left[\frac{1}{16}f_2^2(1-2m^2f_3-2m^2f_4)^2 \overset{+\,m^4f_3f_4}{+} \frac{9}{2500}m^4f_2^2f_3^2+\frac{9}{2500}m^4f_2^2f_4^2-\frac{40\alpha}{E^2}\right]$$

$$\frac{1}{R}\frac{t}{2E}\iint \tau_{xy}^2\,dx\,dy$$

$$\sim\left(\frac{t}{R}\right)\frac{E}{2}\left[\frac{1}{16}f_2^2(1-2m^2f_3-2m^2f_4)^2+m^4f_3f_4+\frac{9}{2500}m^4f_2^2(f_3^2+f_4^2)\right]$$

Therefore the extentional energy

$$\approx \frac{1}{4}f_2^2(1-2m^2f_3-2m^2f_4)^2+2\left(f_3-\frac{m^2}{8}f_2^2\right)^2+2\left(\frac{m^2}{8}f_2^2\right)^2$$
$$+\,4m^4f_3^2f_4^2+\frac{1}{25}m^4f_2^2(f_3^2+f_4^2)+4\left(\frac{\alpha}{E}-\frac{\sigma}{E}\right)^2+\frac{8(1+\nu)\sigma\alpha}{E^2}$$

$$=\frac{1}{4}f_2^2(1+4m^4f_3^2+4m^4f_4^2-4m^2f_3-4m^2f_4+8m^4f_3f_4)$$
$$+\,2\left(f_3^2-\frac{m^2}{4}f_3f_2^2+\frac{m^4}{64}f_2^4\right)+\frac{m^4}{32}f_2^4+4m^4f_3^2f_4^2$$
$$+\,\frac{1}{25}m^4f_2^2(f_3^2+f_4^2)+4\left(\frac{\alpha^2}{E^2}+\frac{\sigma^2}{E^2}\right)+\frac{8\nu}{E^2}\sigma\alpha$$

$$=f_2^2\left(1+\frac{26}{25}m^4f_3^2+\frac{26}{25}m^4f_4^2-\frac{3}{2}m^2f_3-m^2f_4+2m^4f_3f_4+\frac{m^4}{16}f_2^2\right)$$
$$+\,2f_3^2(1+2m^4f_4^2)+4\left[\left(\frac{\alpha}{E}\right)^2+\left(\frac{\sigma}{E}\right)^2\right]+8\nu\left(\frac{\sigma}{E}\right)\left(\frac{\alpha}{E}\right)$$

$$K_x = \frac{\partial^2 w}{\partial x^2} = -\frac{m^2}{R}\left[f_2 \cos\frac{mx}{R}\cos\frac{my}{R} + 4f_3 \cos\frac{2mx}{R}\right]$$

$$K_y = \frac{\partial^2 w}{\partial y^2} = -\frac{m^2}{R}\left[f_2 \cos\frac{mx}{R}\cos\frac{my}{R} + 4f_4 \cos\frac{2my}{R}\right]$$

$$K_{xy} = \frac{\partial^2 w}{\partial x \partial y} = \frac{m^2}{R} f_2 \sin\frac{mx}{R}\sin\frac{my}{R}$$

$$\frac{1}{3}\left(\frac{t}{R}\right)^2 \frac{1}{(1-\nu^2)} m^4 \left[f_2^2 + 8f_3^2 + 8f_4^2\right] = \text{~~Extensional~~ Bending Energy}$$

$$\frac{1}{E}(\sigma_x - \nu\sigma_y) = -\frac{1}{E}(\sigma + \nu\alpha) + \cdots$$

$$-\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 = -\left(\frac{m^2}{8}f_2^2 + m^2 f_4^2\right) \cancel{+ \cdots}$$

Increase in Potential energy $= -8\left(\frac{\sigma}{E}\right)\left[\left(\frac{\sigma}{E} + \nu\frac{\alpha}{E}\right) + m^2\left(\frac{1}{8}f_2^2 + f_4^2\right)\right]$

$$\frac{\alpha}{E} = m^2\left(\frac{1}{8}f_2^2 + f_4^2\right) - f_1 - \nu\frac{\sigma}{E}$$

$$\frac{\sigma}{E} + \nu\frac{\alpha}{E} = (1-\nu^2)\frac{\sigma}{E} + \nu m^2\left(\frac{1}{8}f_2^2 + f_4^2\right) - \nu f_1$$

$$= -8\frac{\sigma}{E}\left[(1-\nu^2)\frac{\sigma}{E} + (1+\nu)m^2\left(\frac{1}{8}f_2^2 + f_4^2\right) - \nu f_1\right]$$

$$4\left[\left(\frac{\alpha}{E}\right)^2+\left(\frac{\sigma}{E}\right)^2+2\nu\left(\frac{\sigma}{E}\right)\left(\frac{\alpha}{E}\right)\right]$$

$$=4\left[m^4\left(\frac{1}{8}f_2^2+f_4^2\right)^2+f_1^2+\cancel{\nu^2\left(\frac{\sigma}{E}\right)^2}-2m^2f_1\left(\frac{1}{8}f_2^2+f_4^2\right)-\cancel{2\nu\frac{\sigma}{E}m^2\left(\frac{1}{8}f_2^2+f_4^2\right)}\right.$$

$$\cancel{+2\nu f_1\frac{\sigma}{E}}$$

$$\left.+\cancel{2\nu m^2\frac{\sigma}{E}\left(\frac{1}{8}f_2^2+f_4^2\right)}-\cancel{2\nu f_1\frac{\sigma}{E}}-\cancel{2}\nu^2\left(\frac{\sigma}{E}\right)^2\right]$$

$$=4\left[m^4\left(\frac{1}{8}f_2^2+f_4^2\right)^2+f_1^2-2m^2f_1\left(\frac{1}{8}f_2^2+f_4^2\right)+(1-\nu^2)\left(\frac{\sigma}{E}\right)^2\right]$$

$$\left.\begin{aligned}&-4(1-\nu^2)\left(\frac{\sigma}{E}\right)^2-8(1+\nu)\frac{\sigma}{E}m^2\left(\frac{1}{8}f_2^2+f_4^2\right)+8\nu\left(\frac{\sigma}{E}\right)f_1\\&+4m^4\left(\frac{1}{8}f_2^2+f_4^2\right)^2+4f_1^2-8m^2f_1\left(\frac{1}{8}f_2^2+f_4^2\right)\end{aligned}\right\}$$

$$\left.\begin{aligned}&8\nu\frac{\sigma}{E}+8f_1-8m^2\left(\frac{1}{8}f_2^2+f_4^2\right)=0\\&\boxed{f_1=m^2\left(\frac{1}{8}f_2^2+f_4^2\right)-\nu\frac{\sigma}{E}}\end{aligned}\right\}\text{Min.}$$

$$-4(1-\nu^2)\left(\frac{\sigma}{E}\right)^2-8(1+\nu)\frac{\sigma}{E}m^2\left(\frac{1}{8}f_2^2+f_4^2\right)+\cancel{8\nu\frac{\sigma}{E}m^2\left(\frac{1}{8}f_2^2+f_4^2\right)}$$

$$-8\nu^2\left(\frac{\sigma}{E}\right)^2+\cancel{4m^4\left(\frac{1}{8}f_2^2+f_4^2\right)^2}+\cancel{4m^4\left(\frac{1}{8}f_2^2+f_4^2\right)^2}+4\nu^2\left(\frac{\sigma}{E}\right)^2$$

$$-\cancel{8m^2\frac{\sigma}{E}\nu\left(\frac{1}{8}f_2^2+f_4^2\right)}-\cancel{8m^4\left(\frac{1}{8}f_2^2+f_4^2\right)^2}+\cancel{8m^2\frac{\sigma}{E}\nu\left(\frac{1}{8}f_2^2+f_4^2\right)}$$

$$W = -4\left(\frac{\sigma}{E}\right)^2 - 8\left(\frac{\sigma}{E}\right) m^2 \left(\frac{1}{8} f_2^2 + f_4^2\right) + f_2^2 \left(1 + \frac{26}{25} m^4 f_3^2 + \frac{26}{25} m^4 f_4^2 - \frac{3}{2} m^2 f_3 - m^2 f_4 + 2m^4 f_3 f_4 + \frac{m^4}{16} f_2^2\right)$$

$$+ 2f_3^2 \left(1 + 2m^4 f_4^2\right) + \frac{1}{3}\left(\frac{t}{R}\right)^2 \frac{1}{(1-\nu^2)} m^4 \left(f_2^2 + 8f_3^2 + 8f_4^2\right)$$

$$\frac{\partial W}{\partial f_2} = 0$$

$$\left(\frac{\sigma}{E}\right) m^2 = 1 + \frac{26}{25} m^4 f_3^2 + \frac{26}{25} m^4 f_4^2 - \frac{3}{2} m^2 f_3 - m^2 f_4 + 2m^4 f_3 f_4 + \frac{m^4}{8} f_2^2 + \frac{1}{3}\left(\frac{t}{R}\right)^2 \frac{m^4}{1-\nu^2}$$

$$\frac{\partial W}{\partial f_3} = 0$$

$$0 = f_2^2 \left(\frac{52}{25} m^2 f_3 - \frac{3}{2} + 2m^2 f_4\right) + \frac{1}{3}\left(\frac{t}{R}\right)^2 \frac{1}{(1-\nu^2)} m^2 16 f_3 + \frac{4 f_3}{m^2}\left(1 + 2m^4 f_4^2\right)$$

$$\frac{\partial W}{\partial f_4} = 0$$

$$2f_4 \, 8\left(\frac{\sigma}{E}\right) = f_2^2 \left(\frac{52}{25} m^2 f_4 - 1 + 2m^2 f_3\right) + 2f_3^2 \cdot 4m^2 f_4 + \frac{1}{3}\left(\frac{t}{R}\right)^2 \frac{m^2}{1-\nu^2} 16 f_4$$

Put $\frac{\sigma}{E} m^2 = \lambda$, and $f_2 m^2 = \alpha$, $f_3 m^2 = \beta$, $f_4 m^2 = \gamma$, $\frac{1}{3}\left(\frac{t}{R}\right)^2 \frac{m^4}{1-\nu^2} = \Theta$

$$\boxed{\begin{aligned} \lambda &= 1 + \frac{26}{25}\beta^2 + \frac{26}{25}\gamma^2 - \frac{3}{2}\beta - \gamma + 2\beta\gamma + \frac{1}{8}\alpha^2 + \Theta \\ 0 &= \alpha^2\left(\frac{52}{25}\beta - \frac{3}{2} + 2\gamma\right) + 4\beta\left(1 + 2\gamma^2\right) + 16\Theta\beta \\ 16\gamma\lambda &= \alpha^2\left(\frac{52}{25}\gamma - 1 + 2\beta\right) + 8\beta^2\gamma + 16\Theta\gamma \end{aligned}}$$

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$$\alpha^2 = 8\left[\lambda - 1 - \frac{26}{25}\beta^2 - \frac{26}{25}\gamma^2 + \frac{3}{2}\beta + \gamma - 2\beta\gamma - \Theta\right]$$

$$0 = 8\left[\lambda - 1 - \frac{26}{25}(\beta^2+\gamma^2) + \frac{3}{2}\beta + \gamma - 2\beta\gamma - \Theta\right]\left(\frac{52}{26}\beta - \frac{3}{2} + 2\gamma\right) + 4\beta(1+2\gamma^2) + 16\Theta\beta$$

$$\gamma\lambda = \frac{1}{2}\left[\lambda - 1 - \frac{26}{25}(\beta^2+\gamma^2) + \frac{3}{2}\beta + \gamma - 2\beta\gamma - \Theta\right]\left(\frac{52}{26}\gamma - 1 + 2\beta\right) + \frac{1}{2}\beta^2\gamma + \Theta\gamma$$

$$\frac{-\beta(1+2\gamma^2) - 4\Theta\beta}{4\gamma\lambda - 2\beta^2\gamma - 4\Theta\gamma} = \frac{\frac{52}{25}\beta - \frac{3}{2} + 2\gamma}{\frac{52}{25}\gamma - 1 + 2\beta}$$

$$\therefore \boxed{\left[\beta(1+2\gamma^2) + 4\Theta\beta\right]\left[\frac{52}{25}\gamma - 1 + 2\beta\right] - \left[4\gamma\lambda - 2\beta^2\gamma - 4\Theta\gamma\right]\left(\frac{52}{25}\beta - \frac{3}{2} + 2\gamma\right) = 0}$$

Θ Known find λ

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$$8\lambda\gamma\left(\frac{52}{25}\beta-\frac{3}{2}+2\gamma\right)-8\gamma\left[1+\frac{26}{25}(\beta^2+\gamma^2)-\frac{3}{2}\beta-\gamma+2\beta\gamma+\Theta\right]\left(\frac{52}{25}\beta-\frac{3}{2}+2\gamma\right)$$

$$+4\beta\gamma(1+2\gamma^2)+16\Theta\beta\gamma=0$$

$$-8\lambda\gamma\left(\frac{52}{25}\beta-\frac{3}{2}+2\gamma\right)+4(\beta^2\gamma+2\Theta\gamma)\left(\frac{52}{25}\beta-\frac{3}{2}+2\gamma\right)+2\left[\beta(1+2\gamma^2)+4\Theta\beta\right]\left(\frac{52}{25}\gamma-1+2\beta\right)=0$$

$$\gamma\left(\frac{52}{25}\beta-\frac{3}{2}+2\gamma\right)\left[4\beta^2+8\Theta-1-\frac{26}{25}(\beta^2+\gamma^2)+\frac{3}{2}\beta+\gamma-2\beta\gamma-\Theta\right]$$

$$+2\left[\beta(1+2\gamma^2)+4\Theta\beta\right]\left[2\gamma+\frac{52}{25}\gamma-1+2\beta\right]=0$$

$$\gamma\left(\frac{52}{25}\beta-\frac{3}{2}+2\gamma\right)\left(\frac{74}{25}\beta^2-\frac{26}{25}\gamma^2-1+\frac{3}{2}\beta+\gamma-2\beta\gamma+7\Theta\right)$$

$$+2\beta\left[(1+2\gamma^2)+4\Theta\right]\left[\frac{102}{25}\gamma-1+2\beta\right]=0.$$

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$$\gamma\left[\frac{52}{25}\beta+\left(2\gamma-\frac{3}{2}\right)\right]\left[\frac{74}{25}\beta^2+\left(\frac{3}{2}-2\gamma\right)\beta+\left(\gamma+7\Theta-\frac{26}{25}\gamma^2-1\right)\right]$$

$$+2\beta\left[(1+2\gamma^2)+4\Theta\right]\left[2\beta+\left(\frac{102}{25}\gamma-1\right)\right]=0$$

$$\begin{array}{r} 74 \\ 52 \\ \hline 148 \\ 370 \\ \hline \end{array}$$

$$\gamma\left[\frac{3848}{625}\beta^3+\left\{\frac{74}{25}\left(2\gamma-\frac{3}{2}\right)+\frac{52}{25}\left(\frac{3}{2}-2\gamma\right)\right\}\beta^2+\left\{\frac{52}{25}\left(\gamma+7\Theta-\frac{26}{25}\gamma^2-1\right)+\left(2\gamma-\frac{3}{2}\right)\left(\frac{3}{2}-2\gamma\right)\right\}\beta\right.$$

$$\left.+\left(2\gamma-\frac{3}{2}\right)\left(\gamma+7\Theta-\frac{26}{25}\gamma^2-1\right)\right]$$

$$+\left[(1+2\gamma^2)+4\Theta\right]\left[4\beta^2+2\left(\frac{102}{25}\gamma-1\right)\beta\right]=0.$$

$$\begin{array}{r} 74 \\ 52 \\ \hline 22 \end{array}$$

$$\left[\frac{3848}{625}\gamma\right]\beta^3+\left[\frac{22}{25}\gamma\left(2\gamma-\frac{3}{2}\right)+4(1+2\gamma^2)+16\Theta\right]\beta^2$$

$$+\left[\frac{52}{25}\left(7\Theta-1-\frac{26}{25}\gamma^2+\gamma\right)\gamma-\left(4\gamma^2-6\gamma+\frac{9}{4}\right)\gamma+2\left(1+2\gamma^2+4\Theta\right)\left(\frac{102}{25}\gamma-1\right)\right]\beta$$

$$+\gamma\left(2\gamma-\frac{3}{2}\right)\left((7\Theta-1)=\frac{26}{25}\gamma^2+\gamma\right)=0$$

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$$\left[\frac{3848}{625}\gamma\right] = A_3$$

$$\left[\frac{244}{25}\gamma^2 - \frac{33}{25}\gamma + 16\Theta + 4\right] = A_2$$

$$\left[\frac{6348}{625}\gamma^3 + \frac{102}{25}\gamma^2 + \left(\frac{236}{5}\Theta + \frac{383}{100}\right)\gamma - 2(1+4\Theta)\right] = A_1$$

$$\gamma\left[-\frac{52}{25}\gamma^3 + \frac{89}{25}\gamma^2 + \left(14\Theta - \frac{7}{2}\right)\gamma - \frac{3}{2}(7\Theta - 1)\right] = A_0$$

$$\underline{A_3\beta^3 + A_2\beta^2 + A_1\beta + A_0 = 0}$$

Let $\Theta = 0.001$ $\qquad$ $\underline{\underline{\gamma = 1.}}$

$A_3 = 6.1568$

$A_2 = 9.76 - 1.32 + 0.016 + 4 = 12.456$

$A_1 = 10.1568 + 4.08 + 0.0472 + 3.83 - 2.008 = 16.1060$

$A_0 = -2.08 + 3.56 + 0.014 - 3.5 + 1.4895 = -0.5165$

$f(\beta) = \beta^3 + 2.02313\beta^2 + 2.61597\beta - 0.0838910 = 0$

$f'(\beta) = 3\beta^2 + 4.04626\beta + 2.61597$

$f(0.031) = -0.0008218$ $\qquad$ $f'(0.031) = 2.74429$

$f(0.0312995) = 0$ $\qquad$ $\underline{\underline{\beta = 0.0312995}}$

$$\beta^2 + 2.05443\beta + 2.68027 = 0$$

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$$\beta = -1.02722 \pm \sqrt{1.02722^2 - 2.68027} \qquad \text{Complex.}$$

$$\frac{\beta\left[(1+2\gamma^2)+4\Theta\right]\left[\frac{52}{25}\gamma-1+2\beta\right]}{\gamma\left[\frac{52}{25}\beta-\frac{3}{2}+2\gamma\right]} + (2\beta^2+4\Theta) = 4\lambda$$

$$\lambda = \frac{1}{4}\left[\frac{0.0312995 \times 3.004 \times 1.14260}{0.5312995} + 0.0059593\right] = 0.052041$$

$$\frac{\sigma}{E} m^2 = 0.052041 \qquad \text{Let } m = 12$$

$$\frac{1}{3(1-\nu^2)}\left(\frac{t}{R} m^2\right)^2 = 0.001 \qquad \left(\frac{R}{t}\right)^2 = \frac{144^2\, 10^3}{3(1-\nu^2)}$$

$$\frac{\sigma}{E} = 0.0003614 \qquad \left(\frac{R}{t}\right) = \frac{144 \times 31.62278}{1.65227}$$

$$= 2756$$

$$\frac{\sigma}{E}\frac{R}{t} = \underline{0.996} \quad !!!$$

$$\alpha^2 = 8\left[0.052041 \times 1 - \frac{26}{25}(1.00271) + 0.078062 \times 1 - 1.04082 - 0.001\right]$$

$$= (-)$$

Impossible !!!

$\gamma = -1$

$A_3 = -6.1568$

$A_2 = 9.76 + 1.32 + 0.016 + 4 = 15.096$

$A_1 = -10.1568 + 4.08 - 0.0472 - 3.83 - 2.008 = -11.962$

$A_0 = -2.08 - 3.56 + 0.014 - 3.5 - 1.4895 = -10.6155$

$f(\beta) = \beta^3 - 2.45192\,\beta^2 + 1.94289\beta + 1.72419 = 0$

$f'(\beta) = 3\beta^2 - 4.90384\,\beta + 1.94289$

$\cancel{f(-\beta) = \beta^3 + 2.45192\,\beta^2 + 1.94289\,\beta - 1.72419}$

$\cancel{f'(-\beta) = 3\beta^3 + 4.90384\,\beta + 1.94289}$

$f(0.50) = -0.01476$ $\quad$ $f'(0.50) = 5.1448$

$f(0.50287) = +0.00004$ $\quad$ $f'(0.50287) = 5.16$

$\beta = -0.50286$

$\beta^2 - 2.95478\,\beta + 3.42873 = 0$

$\beta = 1.47738 \pm \sqrt{1.47738^2 - 3.42873}$ Complex

$m^2 f_1 = \delta$

$\boxed{\delta = \frac{1}{8}\alpha^2 + \gamma^2 - 4\lambda}$

$\boxed{6.03}$

$$\lambda\ \frac{1}{4}\left[\frac{0.50286 \times 3.004 \times (-2.07428)}{-5.58000} + 0.25687\right]$$

$=$ too Big !!!

Lot of Mistakes!!!

If $\alpha = 0$

$$0 = 1 + 2\gamma^2 + 4\Theta$$

$$2\lambda = \beta^2 + 2\Theta$$

$$\lambda = \Theta$$

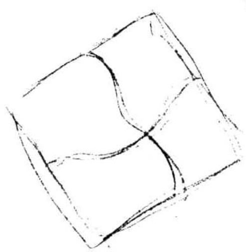
6.6
1.4
5.2

thus

$$\frac{\sigma}{E} m^2 = \frac{1}{3(1-\nu^2)}\left(\frac{t}{R} m^2\right)^2$$

(5.0)

$$\boxed{\frac{\sigma}{E}\frac{R}{t} = \frac{1}{3(1-\nu^2)}\left(\frac{t}{R} m^2\right)}$$



$$\frac{3(1-\nu^2)}{\left(\frac{t}{R}\right)^2}\frac{\sigma}{E} = m^2$$

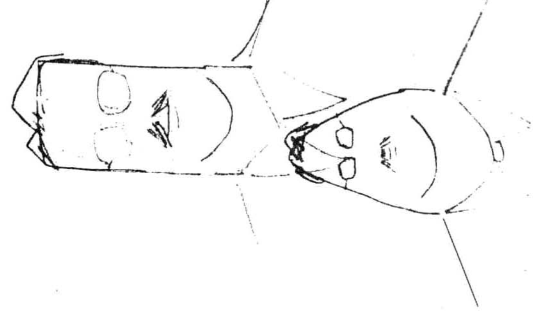
0.1

1.44

0.144

0.05

$w = f_1 + f_2 ($



$$\frac{w}{R} = f_0 + f_1 \cos^2 \frac{m(x+y)}{2R} \cos^2 \frac{m(x-y)}{2R}$$

$$= f_0 + f_1 \left\{ \cos^2 \frac{mx}{2R} \cos^2 \frac{my}{2R} - \sin^2 \frac{mx}{2R} \sin^2 \frac{my}{2R} \right\}^2$$

$$= f_0 + f_1 \left\{ 1 - \sin^2 \frac{mx}{2R} - \sin^2 \frac{my}{2R} \right\}^2$$

$$= f_0 + \frac{1}{4} f_1 \left\{ \cos \frac{mx}{R} + \cos \frac{my}{R} \right\}^2$$

$$= f_0 + \frac{1}{4} f_1 \left\{ \cos^2 \frac{mx}{R} + 2 \cos \frac{mx}{R} \cos \frac{my}{R} + \cos^2 \frac{my}{R} \right\}$$

$$= f_0 + \frac{1}{4} f_1 \left\{ \frac{1}{2}\left(1 + \cos \frac{2mx}{R}\right) + 2 \cos \frac{mx}{R} \cos \frac{my}{R} + \frac{1}{2}\left(1 + \cos \frac{2my}{R}\right) \right\}$$

$$= \left(f_0 + \frac{1}{4} f_1\right) + \frac{1}{8} f_1 \cos \frac{2mx}{R} + \frac{1}{8} f_1 \cos \frac{2my}{R} + \frac{1}{2} f_1 \cos \frac{mx}{R} \cos \frac{my}{R}$$

$$= \left(f_0 + \frac{1}{4} f_1\right) + \frac{1}{2} f_1 \left[\cos \frac{mx}{R} \cos \frac{my}{R} + \frac{1}{4} \cos \frac{2mx}{R} + \frac{1}{4} \cos \frac{2my}{R} \right]$$

$$\frac{1}{R} \frac{\partial^2 w}{\partial x^2} = -\left(\frac{m}{R}\right)^2 \frac{1}{2} f_1 \left[\cos \frac{mx}{R} \cos \frac{my}{R} + \cos \frac{2mx}{R} \right]$$

$$\frac{1}{R} \frac{\partial^2 w}{\partial y^2} = -\left(\frac{m}{R}\right)^2 \frac{1}{2} f_1 \left[\cos \frac{mx}{R} \cos \frac{my}{R} + \cos \frac{2my}{R} \right]$$

$$\frac{1}{R} \frac{\partial^2 w}{\partial x \partial y} = \left(\frac{m}{R}\right)^2 \frac{1}{2} f_1 \left[\sin \frac{mx}{R} \sin \frac{my}{R} \right]$$

$$\Delta\Delta F = E\frac{m^2}{R^2}\frac{1}{2}f_1\Big[\left(\frac{1}{2}f_1 m^2\right)\Big\{-\frac{1}{2}\cos\frac{2mx}{R}-\frac{1}{2}\cos\frac{2my}{R}$$ 6a5

$$-\frac{1}{2}\left(\cos\frac{3mx}{R}+\cos\frac{mx}{R}\right)\cos\frac{my}{R}-\frac{1}{2}\cos\frac{mx}{R}\left(\cos\frac{3my}{R}+\cos\frac{my}{R}\right)$$

$$-\cos\frac{2mx}{R}\cos\frac{2my}{R}\Big\}+\cos\frac{mx}{R}\cos\frac{my}{R}+\cos\frac{2mx}{R}\Big]$$

$$=E\left(\frac{m}{R}\right)^2\frac{f_1}{2}\Big[\left(1-\frac{f_1 m^2}{2}\right)\cos\frac{mx}{R}\cos\frac{my}{R}+\left(1-\frac{f_1 m^2}{4}\right)\cos\frac{2mx}{R}$$

$$-\frac{f_1 m^2}{4}\cos\frac{2my}{R}-\frac{f_1 m^2}{2}\cos\frac{2mx}{R}\cos\frac{2my}{R}-\frac{f_1 m^2}{4}\cos\frac{3mx}{R}\cos\frac{my}{R}$$

$$-\frac{f_1 m^2}{4}\cos\frac{mx}{R}\cos\frac{3my}{R}\Big]$$

$$F=E\left(\frac{R}{m}\right)^2\frac{f_1}{2}\Big[\frac{1}{4}\left(1-\frac{f_1 m^2}{2}\right)\cos\frac{mx}{R}\cos\frac{my}{R}+\frac{1}{16}\left(1-\frac{f_1 m^2}{4}\right)\cos\frac{2mx}{R}$$

$$-\frac{f_1 m^2}{64}\cos\frac{2my}{R}-\frac{f_1 m^2}{128}\cos\frac{2mx}{R}\cos\frac{2my}{R}-\frac{f_1 m^2}{400}\cos\frac{3mx}{R}\cos\frac{my}{R}$$

$$-\frac{f_1 m^2}{400}\cos\frac{mx}{R}\cos\frac{3my}{R}\Big]-\frac{\sigma}{2}y^2+\frac{\lambda}{2}x^2$$

$$\sigma_x=E\frac{f_1}{2}\Big[\frac{1}{4}\left(\frac{f_1 m^2}{2}-1\right)\cos\frac{mx}{R}\cos\frac{my}{R}+\frac{f_1 m^2}{16}\cos\frac{2my}{R}+\frac{f_1 m^2}{32}\cos\frac{2mx}{R}\cos\frac{2my}{R}$$

$$+\frac{f_1 m^2}{400}\cos\frac{3mx}{R}\cos\frac{my}{R}+\frac{9f_1 m^2}{400}\cos\frac{mx}{R}\cos\frac{3my}{R}\Big]-\sigma$$

$$\sigma_y = E\frac{f_1}{2}\Big[\frac{1}{4}\Big(\frac{f_1 m^2}{2}-1\Big)\cos\frac{mx}{R}\cos\frac{my}{R}+\frac{1}{4}\Big(\frac{f_1 m^2}{4}-1\Big)\cos\frac{2mx}{R} \qquad \underline{606}$$

$$+\frac{f_1 m^2}{32}\cos\frac{2mx}{R}\cos\frac{2my}{R}+\frac{9f_1 m^2}{400}\cos\frac{3mx}{R}\cos\frac{my}{R}+\frac{f_1 m^2}{400}\cos\frac{mx}{R}\cos\frac{3my}{R}\Big]+\lambda$$

$$\tau_{xy} = E\frac{f_1}{2}\Big[\frac{1}{4}\Big(\frac{f_1 m^2}{2}-1\Big)\sin\frac{mx}{R}\sin\frac{my}{R}+\frac{f_1 m^2}{32}\sin\frac{2mx}{R}\sin\frac{2my}{R}$$

$$+\frac{3}{400}f_1 m^2\sin\frac{3mx}{R}\sin\frac{my}{R}+\frac{3f_1 m^2}{400}\sin\frac{mx}{R}\sin\frac{3my}{R}\Big]$$

$$\frac{1}{E}(\sigma_y-\nu\sigma_x)=\frac{1}{E}(\lambda+\nu\sigma)+\frac{f_1}{2}\Big[\frac{1-\nu}{4}\Big(\frac{f_1 m^2}{2}-1\Big)\cos\frac{mx}{R}\cos\frac{my}{R}+\frac{1}{4}\Big(\frac{f_1 m^2}{4}-1\Big)\cos\frac{2mx}{R}$$

$$-\nu\frac{f_1 m^2}{16}\cos\frac{2my}{R}+\frac{1-\nu}{32}f_1 m^2\cos\frac{2mx}{R}\cos\frac{2my}{R}+\frac{9-\nu}{400}f_1 m^2\cos\frac{3mx}{R}\cos\frac{my}{R}$$

$$+\frac{1-9\nu}{400}f_1 m^2\cos\frac{mx}{R}\cos\frac{3my}{R}\Big]$$

$$-\frac{1}{2}\Big(\frac{\partial w}{\partial y}\Big)^2=\frac{f_1}{2}\Big[-\frac{f_1 m^2}{4}\Big(\cos\frac{mx}{R}\sin\frac{my}{R}+\frac{1}{2}\sin\frac{2my}{R}\Big)^2\Big]$$

$$=\frac{f_1}{2}\Big[-\frac{f_1 m^2}{4}\Big\{\cos^2\frac{mx}{R}\sin^2\frac{my}{R}+\frac{1}{2}\cos\frac{mx}{R}\Big(\cos\frac{my}{R}-\cos\frac{3my}{R}\Big)$$

$$+\frac{1}{8}\Big(1-\cos\frac{4my}{R}\Big)\Big\}$$

$$=\frac{f_1}{2}\Big[-\frac{f_1 m^2}{4}\Big\{\frac{1}{4}\Big(1+\cos\frac{2mx}{R}\Big)\Big(1-\cos\frac{2my}{R}\Big)+\frac{1}{2}\cos\frac{mx}{R}\cos\frac{my}{R}$$

$$-\frac{1}{2}\cos\frac{mx}{R}\cos\frac{3my}{R}+\frac{1}{8}-\frac{1}{8}\cos\frac{4my}{R}\Big]$$

$$=\frac{f_1}{2}\Big[-\frac{f_1 m^2}{4}\Big\{\frac{3}{8}+\frac{1}{4}\cos\frac{2mx}{R}-\frac{1}{4}\cos\frac{2my}{R}-\frac{1}{4}\cos\frac{2mx}{R}\cos\frac{2my}{R}$$

$$+\frac{1}{2}\cos\frac{mx}{R}\cos\frac{my}{R}-\frac{1}{2}\cos\frac{mx}{R}\cos\frac{3my}{R}-\frac{1}{8}\cos\frac{4my}{R}\Big\}\Big]$$

$$\frac{1}{4}f_1 + \frac{1}{E}(\lambda + \nu\sigma) + f_0 - \frac{3}{64} f_1 (f_1 m^2) = 0$$

$$\boxed{\frac{\lambda}{E} = \frac{3}{64} f_1 (f_1 m^2) - \left(f_0 + \frac{f_1}{4}\right) - \nu\frac{\sigma}{E}}$$

The increase in potential energy

$$-\left[+\frac{1}{E}(\sigma + \nu\lambda) + \frac{3}{64} f_1 (f_1 m^2)\right] 8\frac{\sigma}{E}$$

$$\boxed{\wp_1 = -8\frac{\sigma}{E}\left[(1-\nu^2)\frac{\sigma}{E} + (1+\nu)\frac{3}{64} f_1 (f_1 m^2) - \nu\left(f_0 + \frac{f_1}{4}\right)\right]}$$

$$\sigma_x + \sigma_y = (-\sigma + \lambda) + E\frac{f_1}{2}\left[\frac{1}{2}\left(\frac{f_1 m^2}{2} - 1\right)\cos\frac{mx}{R}\cos\frac{my}{R} + \frac{1}{4}\left(\frac{f_1 m^2}{4} - 1\right)\cos\frac{2mx}{R}\right.$$
$$+ \frac{f_1 m^2}{16}\cos\frac{2my}{R} + \frac{f_1 m^2}{16}\cos\frac{2mx}{R}\cos\frac{2mx}{R} + \frac{1}{40} f_1 m^2 \cos\frac{3mx}{R}\cos\frac{my}{R}$$
$$\left. + \frac{1}{40} f_1 m^2 \cos\frac{mx}{R}\cos\frac{3my}{R}\right]$$

$$\wp_2 = 4\left(\frac{\sigma - \lambda}{E}\right)^2 + \frac{f_1^2}{4}\left[\frac{1}{4}\left(\frac{f_1 m^2}{2} - 1\right)^2 + \frac{1}{8}\left(\frac{f_1 m^2}{4} - 1\right)^2 + \frac{1}{128}(f_1 m^2)^2\right.$$
$$\left. + \frac{(f_1 m^2)^2}{256} + \frac{2(f_1 m^2)^2}{1600}\right]$$

$$= 4\left(\frac{\sigma - \lambda}{E}\right)^2 + \frac{f_1^2}{16}\left[\frac{(f_1 m^2)^2}{4} - (f_1 m^2) + 1 + \frac{(f_1 m^2)^2}{32} - \frac{1}{4}(f_1 m^2) + \frac{1}{2}\right.$$
$$\left. + \left(\frac{1}{32} + \frac{1}{64} + \frac{1}{200}\right)(f_1 m^2)^2\right]$$

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$$\beta_2 = 4\left(\frac{\sigma-\lambda}{E}\right)^2 + \frac{f_1^2}{16}\left[\left(\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\frac{1}{200}\right)(f_1 m^2)^2 - \frac{5}{4}(f_1 m^2) + \frac{3}{2}\right]$$

$$= 4\left(\frac{\sigma-\lambda}{E}\right)^2 + \frac{f_1^2}{16}\left[\frac{533}{1600}(f_1 m^2)^2 - \frac{5}{4}(f_1 m^2) + \frac{3}{2}\right] + 8(1+\nu)\frac{\sigma\lambda}{E}$$

$$\boxed{\beta_2 = 4\left(\frac{\sigma-\lambda}{E}\right)^2 + \frac{f_1^2}{64}\left[\frac{533}{400}(f_1 m^2)^2 - 5(f_1 m^2) + 6\right] + 8(1+\nu)\frac{\sigma\lambda}{E}}$$

$$\beta_3 = \frac{1}{12(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4 \frac{f_1^2}{4}\left[4+2+2\right]$$

$$\boxed{\beta_3 = \frac{1}{6(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4 f_1^2}$$

$$4\left(\frac{\sigma-\lambda}{E}\right)^2 = 4\left\{(1+\nu)\frac{\sigma}{E} - \frac{3}{64} f_1 (f_1 m^2) + \left(f_0 + \frac{f_1}{4}\right)\right\}^2$$

$$= 4\left\{(1+\nu)^2\left(\frac{\sigma}{E}\right)^2 - \frac{3}{32}(1+\nu)\frac{\sigma}{E} f_1 (f_1 m^2) + 2(1+\nu)\left(f_0+\frac{f_1}{4}\right)\frac{\sigma}{E}\right.$$

$$\left. + \frac{9}{64^2} f_1^2 (f_1 m^2)^2 + \left(f_0 + \frac{f_1}{4}\right)^2 - \frac{3}{32} f_1 (f_1 m^2)\left(f_0 + \frac{f_1}{4}\right)\right\}$$

$$\beta_1 + 4\left(\frac{\sigma-\lambda}{E}\right)^2 = 4\left(\frac{\sigma}{E}\right)^2\left\{(1+\nu)^2 - 2(1-\nu^2)\right\} - \frac{3}{4}(1+\nu)\frac{\sigma}{E} f_1 (f_1 m^2)$$

$$+ \frac{\sigma}{E} 8(1+\nu)\left(f_0 + \frac{f_1}{4}\right) + \frac{9}{1024} f_1^2 (f_1 m^2)^2 + 4\left(f_0 + \frac{f_1}{4}\right)^2 - \frac{3}{8} f_1 (f_1 m^2)\left(f_0 + \frac{f_1}{4}\right)$$

$64 \times 16 = 384$

$1+\nu - 2 + 2\nu - \nu$

$$8(1+\nu)\frac{\sigma\lambda}{E} = 8(1+\nu)\frac{\sigma}{E}\left[\frac{3}{64}f_1(f_1m^2) - (f_0+\frac{f_1}{4}) - 4\frac{\sigma}{E}\right]$$

$$\wp_1 + 4\left(\frac{\sigma-\lambda}{E}\right)^2 + 2(1+\nu)4\frac{\sigma\lambda}{E} = K$$

$$= 4\left(\frac{\sigma}{E}\right)^2\left\{(1+\nu)^2 - 2(1-\nu^2) - 2\nu(1+\nu)\right\} - \frac{3}{8}(1+\nu)\frac{\sigma}{E}f_1(f_1m^2) + 8\nu\frac{\sigma}{E}(f_0+\frac{f_1}{4})$$

$$+ \frac{9}{1024}f_1^2(f_1m^2)^2 + 4(f_0+\frac{f_1}{4})^2 - \frac{3}{8}f_1(f_1m^2)(f_0+\frac{f_1}{4})$$

$\frac{\partial K}{\partial f_0} = 0$ gives $\boxed{\nu\frac{\sigma}{E} + (f_0+\frac{f_1}{4}) - \frac{3}{64}f_1(f_1m^2) = 0}$

$$\therefore \quad K = -4\left(\frac{\sigma}{E}\right)^2(1-\nu^2) - \frac{3}{8}(1+\nu)\frac{\sigma}{E}f_1(f_1m^2) + \frac{9}{1024}f_1^2(f_1m^2)^2 - 4(f_0+\frac{f_1}{4})^2$$

But $\quad 4(f_0+\frac{f_1}{4})^2 = \left\{2\nu\frac{\sigma}{E} - \frac{3}{32}f_1(f_1m^2)\right\}^2$

$$= 4\left(\frac{\sigma}{E}\right)^2\nu^2 - \frac{3}{8}\nu\frac{\sigma}{E}f_1(f_1m^2) + \frac{9}{1024}f_1^2(f_1^2m^4)$$

32
32
64
96
1024

$$\therefore \quad K = -4\left(\frac{\sigma}{E}\right)^2 - \frac{3}{8}\left(\frac{\sigma}{E}\right)f_1(f_1m^2)$$

The potential of the System.

$$P = -4\left(\frac{\sigma}{E}\right)^2 - \frac{3}{8}\left(\frac{\sigma}{E}\right)f_1(f_1m^2) + \frac{f_1^2}{64}\left[\frac{533}{400}(f_1m^2)^2 - 5(f_1m^2) + 6\right] + \frac{1}{6(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4 f_1^2$$

$$\frac{\partial P}{\partial f_1} = 0$$

$$\frac{3}{4}\left(\frac{\sigma}{E}\right)m^2 = \frac{1}{32}\left[\frac{533}{200}(f_1m^2)^2 - 7.5(f_1m^2) + 6\right] + \frac{1}{3(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4$$

$$\frac{\sigma}{E} = \left[\frac{533}{4800} f_1^2 m^2 - \frac{15}{48} f_1 + \frac{1}{4} \frac{1}{m^2} \right] + \frac{4}{9(1-\nu^2)} \left(\frac{t}{R}\right)^2 m^2$$

$$= \left\{ \frac{533}{4800} f_1^2 + \frac{4}{9(1-\nu^2)} \left(\frac{t}{R}\right)^2 \right\} m^2 + \frac{1}{4} \frac{1}{m^2} - \frac{5}{16} f_1$$

$$= 2 \left\{ \frac{533}{12 \times (40)^2} f_1^2 + \frac{1}{9(1-\nu^2)} \left(\frac{t}{R}\right)^2 \right\}^{\frac{1}{2}} - \frac{5}{16} f_1$$

$$\boxed{\frac{\sigma}{E} \frac{R}{t} = \left\{ \frac{533}{3 \times (40)^2} \left(\frac{\delta}{t}\right)^2 + \frac{4}{9(1-\nu^2)} \right\}^{\frac{1}{2}} - \frac{5}{16} \left(\frac{\delta}{t}\right)}$$

$$\frac{\partial\left(\frac{\sigma}{E}\frac{R}{t}\right)}{\partial\left(\frac{\delta}{t}\right)} = 0, \qquad \frac{\frac{1}{2} \times 2 \, \frac{533}{3 \times 1600} \left(\frac{\delta}{t}\right)}{\left\{ \frac{533}{3 \times 1600} \left(\frac{\delta}{t}\right)^2 + \frac{4}{9(1-\nu^2)} \right\}^{\frac{1}{2}}} = \frac{5}{16}$$

$$\left(\frac{533}{4800}\right)^2 \left(\frac{\delta}{t}\right)^2 = \frac{25}{256} \left\{ \frac{533}{4800} \left(\frac{\delta}{t}\right)^2 + \frac{4}{9(1-\nu^2)} \right\}$$

$$\frac{533}{4800} \left(\frac{533}{4800} - \frac{25}{256} \right) \left(\frac{\delta}{t}\right)^2 = \frac{25}{9 \times 64 (1-\nu^2)}$$

$$\frac{533}{75} \times \frac{1}{64} \left(\frac{533}{75} - \frac{25}{4} \right) \left(\frac{\delta}{t}\right)^2 = \frac{25}{9(1-\nu^2)}$$

$$\frac{533}{\underset{25}{\cancel{75}}} \times \frac{1}{64} \times \frac{1}{\underset{25}{\cancel{75}}} \times \frac{1}{4} (257) \left(\frac{\delta}{t}\right)^2 = \frac{25}{\cancel{9}(1-\nu^2)}$$

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$$\left(\frac{\delta}{t}\right)^2 = \frac{1}{1-\nu^2} \; \frac{62500 \times 64}{533 \times 257} = \frac{4000000}{124652.71}$$

$$\left(\frac{\delta}{t}\right)^2 = 32.0892 \qquad \frac{\delta}{t} = 5.6648$$

$$\left(\frac{\sigma}{E}\frac{R}{t}\right)_{min} = 5.6648 \left\{ \frac{533}{\cancel{4800}_{300}} \; \frac{\cancel{16}}{5} - \frac{5}{16} \right\}$$

$$= 5.6648 \left\{ \frac{5.33}{15} - \frac{5}{16} \right\} = 5.6648 \left\{ 0.355333 - 0.312500 \right\}$$

$$= \underline{\underline{0.24264}} \qquad !!!$$

$$\frac{0.24264}{0.606} = \underline{0.400}$$

$$\left(\frac{\delta}{t}\right) = 18.02912$$

$$\frac{\sigma R}{E t} = \left\{ \frac{5330}{4800} (18.02912)^2 + \frac{4}{8.19} \right\}^{\frac{1}{2}} - \frac{5}{16} \times 18.02912$$

$$= \left(36.09401 + 0.48840 \right)^{\frac{1}{2}} - 5.6341$$

$$= \underline{\underline{0.41426}} \qquad !!!$$

$$\frac{w}{R} = f_0 + f_1\left[\cos^4 \frac{m(x+y)}{2R} \cos^2 \frac{m(x-y)}{2R} \right]$$

$$= f_0 + f_1\left[\frac{1}{16}\left\{ \cos\frac{mx}{R} + \cos\frac{my}{R} \right\}^4 \right]$$

$$= f_0 + f_1\left[\frac{1}{16}\left\{ \cos^2\frac{mx}{R} + 2\cos\frac{mx}{R}\cos\frac{my}{R} + \cos^2\frac{my}{R} \right\}^2 \right]$$

$$= f_0 + f_1\left[\frac{1}{16}\left\{ 1 + \frac{1}{2}\cos\frac{2mx}{R} + \frac{1}{2}\cos\frac{2my}{R} + 2\cos\frac{mx}{R}\cos\frac{my}{R} \right\}^2 \right]$$

$$= f_0 + f_1\Big[\frac{1}{16}\Big\{ 1 + \frac{1}{4}\cos^2\frac{2mx}{R} + \frac{1}{4}\cos^2\frac{2my}{R} + 4\cos^2\frac{mx}{R}\cos^2\frac{my}{R}$$

$$+ \cos\frac{2mx}{R} + \cos\frac{2my}{R} + 4\cos\frac{mx}{R}\cos\frac{my}{R} + \frac{1}{2}\cos\frac{2mx}{R}\cos\frac{2my}{R}$$

$$+ \left(\cos\frac{3mx}{R} + \cos\frac{mx}{R}\right)\cos\frac{my}{R} + \cos\frac{mx}{R}\left(\cos\frac{3my}{R} + \cos\frac{my}{R}\right)\Big\}\Big]$$

$$= f_0 + f_1\Big[\frac{1}{16}\Big\{ 1 + \frac{1}{8} + \frac{1}{8}\cos\frac{4mx}{R} + \frac{1}{8} + \frac{1}{8}\cos\frac{4my}{R}$$

$$+ 1 + \cos\frac{2mx}{R} + \cos\frac{2my}{R} + \cos\frac{2mx}{R}\cos\frac{2my}{R} + \cos\frac{2mx}{R} + \cos\frac{2my}{R}$$

$$+ 6\cos\frac{mx}{R}\cos\frac{my}{R} + \frac{1}{2}\cos\frac{2mx}{R}\cos\frac{2my}{R} + \cos\frac{3mx}{R}\cos\frac{my}{R} + \cos\frac{mx}{R}\cos\frac{3my}{R}\Big\}\Big]$$

$$\frac{w}{R} = f_0 + f_1\Big[\frac{9}{64} + \frac{3}{8}\cos\frac{mx}{R}\cos\frac{my}{R} + \frac{1}{8}\cos\frac{2mx}{R} + \frac{1}{8}\cos\frac{2my}{R}$$

$$+ \frac{3}{32}\cos\frac{2mx}{R}\cos\frac{2my}{R} + \frac{1}{16}\cos\frac{3mx}{R}\cos\frac{my}{R} + \frac{1}{16}\cos\frac{mx}{R}\cos\frac{3my}{R}$$

$$+ \frac{1}{128}\cos\frac{4mx}{R} + \frac{1}{128}\cos\frac{4my}{R}\Big]$$

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$$\frac{w}{R} = \left(f_0 + \frac{9}{64} f_1\right) + \frac{f_1}{8}\Big[3\cos\frac{mx}{R}\cos\frac{my}{R} + \cos\frac{2mx}{R} + \cos\frac{2my}{R}$$

$$+ \frac{3}{4}\cos\frac{2mx}{R}\cos\frac{2my}{R} + \frac{1}{2}\cos\frac{3mx}{R}\cos\frac{my}{R} + \frac{1}{2}\cos\frac{mx}{R}\cos\frac{3my}{R}$$

$$+ \frac{1}{16}\cos\frac{4mx}{R} + \frac{1}{16}\cos\frac{4my}{R}\Big]$$

$$\left(\frac{\partial w}{\partial y}\right)_0 = m\frac{f_1}{8}\Big[3\cos\frac{mx}{R}\sin\frac{my}{R} + 2\sin\frac{2my}{R} + \frac{3}{2}\cos\frac{2mx}{R}\sin\frac{2my}{R}$$

$$+ \frac{1}{2}\cos\frac{3mx}{R}\sin\frac{my}{R} + \frac{3}{2}\cos\frac{mx}{R}\sin\frac{3my}{R} + \frac{1}{4}\sin\frac{4my}{R}\Big]$$

$$\frac{1}{R}\frac{\partial^2 w}{\partial x^2} = -\left(\frac{m}{R}\right)^2\frac{f_1}{8}\Big[3\cos\frac{mx}{R}\cos\frac{my}{R} + 4\cos\frac{2mx}{R} + 3\cos\frac{2mx}{R}\cos\frac{2my}{R}$$

$$+ \frac{9}{2}\cos\frac{3mx}{R}\cos\frac{my}{R} + \frac{1}{2}\cos\frac{mx}{R}\cos\frac{3my}{R} + \cos\frac{4mx}{R}\Big]$$

$$\frac{1}{R}\frac{\partial^2 w}{\partial y^2} = -\left(\frac{m}{R}\right)^2\frac{f_1}{8}\Big[3\cos\frac{mx}{R}\cos\frac{my}{R} + 4\cos\frac{2my}{R} + 3\cos\frac{2mx}{R}\cos\frac{2my}{R}$$

$$+ \frac{1}{2}\cos\frac{3mx}{R}\cos\frac{my}{R} + \frac{9}{2}\cos\frac{mx}{R}\cos\frac{3my}{R} + \cos\frac{4my}{R}\Big]$$

$$\frac{1}{R}\frac{\partial^2 w}{\partial x\partial y} = +\left(\frac{m}{R}\right)^2\frac{f_1}{8}\Big[3\sin\frac{mx}{R}\sin\frac{my}{R} + 3\sin\frac{2mx}{R}\sin\frac{2my}{R} + \frac{3}{2}\sin\frac{3mx}{R}\sin\frac{my}{R}$$

$$+ \frac{3}{2}\sin\frac{mx}{R}\sin\frac{3my}{R}\Big]$$

$$\frac{3}{2} + \frac{27}{2}$$

$$\Delta\Delta F=\left(\frac{m}{R}\right)^2 E\,\frac{f_1}{8}\Big[\Big(\frac{f_1}{8}m^2\Big)\Big\{\,9\Big(\sin^2\frac{mx}{R}\sin^2\frac{my}{R}-\cos^2\frac{mx}{R}\cos^2\frac{my}{R}\Big)$$

$$+9\Big(\sin^2\frac{2mx}{R}\sin^2\frac{2my}{R}-\cos^2\frac{2mx}{R}\cos^2\frac{2my}{R}\Big)$$

$$+\frac{9}{4}\Big(\sin^2\frac{3mx}{R}\sin^2\frac{my}{R}-\cos^2\frac{3mx}{R}\cos^2\frac{mx}{R}\Big)$$

$$+\frac{9}{4}\Big(\sin^2\frac{mx}{R}\sin^2\frac{3my}{R}-\cos^2\frac{mx}{R}\cos^2\frac{3my}{R}\Big)$$

$$+18\Big(\sin\frac{mx}{R}\sin\frac{2mx}{R}\sin\frac{my}{R}\sin\frac{2my}{R}-\cos\frac{mx}{R}\cos\frac{2mx}{R}\cos\frac{my}{R}\cos\frac{2my}{R}\Big)$$

$$+3\Big(3\sin\frac{mx}{R}\sin\frac{3mx}{R}\sin^2\frac{my}{R}-5\cos\frac{mx}{R}\cos\frac{3mx}{R}\cos^2\frac{my}{R}\Big)$$

$$+3\Big(3\sin^2\frac{mx}{R}\sin\frac{my}{R}\sin\frac{3my}{R}-5\cos^2\frac{mx}{R}\cos\frac{my}{R}\cos\frac{3my}{R}\Big)$$

$$+3\Big(3\sin\frac{2mx}{R}\sin\frac{3mx}{R}\sin\frac{my}{R}\sin\frac{2my}{R}-5\cos\frac{2mx}{R}\cos\frac{3mx}{R}\cos\frac{my}{R}\cos\frac{2my}{R}\Big)$$

$$+3\Big(3\sin\frac{mx}{R}\sin\frac{2mx}{R}\sin\frac{2my}{R}\sin\frac{3my}{R}-5\cos\frac{mx}{R}\cos\frac{2mx}{R}\cos\frac{2my}{R}\cos\frac{3my}{R}\Big)$$

$$+9\Big(\sin\frac{mx}{R}\sin\frac{3mx}{R}\sin\frac{my}{R}\sin\frac{3my}{R}-\cos\frac{mx}{R}\cos\frac{3mx}{R}\cos\frac{my}{R}\cos\frac{3my}{R}\Big)$$

$$\mp 12\cos\frac{mx}{R}\cos\frac{my}{R}\cos\frac{2my}{R}\mp 16\cos\frac{2mx}{R}\cos\frac{2my}{R}\mp 12\cos\frac{2mx}{R}\cos^2\frac{2my}{R}$$

$$\mp 18\cos\frac{3mx}{R}\cos\frac{my}{R}\cos\frac{2my}{R}\mp 2\cos\frac{mx}{R}\cos\frac{2my}{R}\cos\frac{3my}{R}\mp 4\cos\frac{4mx}{R}\cos\frac{3my}{R}$$

$$\mp 3\cos\frac{mx}{R}\cos\frac{my}{R}\cos\frac{4my}{R}\mp 4\cos\frac{2mx}{R}\cos\frac{4my}{R}\mp 3\cos\frac{2mx}{R}\cos\frac{2my}{R}\cos\frac{4my}{R}$$

$$\mp\frac{9}{2}\cos\frac{3mx}{R}\cos\frac{my}{R}\cos\frac{4my}{R}\mp\frac{1}{2}\cos\frac{mx}{R}\cos\frac{3my}{R}\cos\frac{4my}{R}\mp\cos\frac{4mx}{R}\cos\frac{4my}{R}$$

$$\mp 12\cos\frac{m\pi x}{R}\cos\frac{2m\pi x}{R}\cos\frac{n\pi y}{R} \mp 16\cos\frac{2m\pi x}{R}\cos\frac{2n\pi y}{R} \mp 12\cos^2\frac{2m\pi x}{R}\cos\frac{2n\pi y}{R}$$ 615

$$\mp 2\cos\frac{2m\pi x}{R}\cos\frac{3m\pi x}{R}\cos\frac{n\pi y}{R} \mp 18\cos\frac{m\pi x}{R}\cos\frac{2m\pi x}{R}\cos\frac{3n\pi y}{R} \mp 4\cos\frac{2m\pi x}{R}\cos\frac{4n\pi y}{R}$$

$$\mp 3\cos\frac{m\pi x}{R}\cos\frac{4m\pi x}{R}\cos\frac{n\pi y}{R} \mp 4\cos\frac{4m\pi x}{R}\cos\frac{2n\pi y}{R} \mp 3\cos\frac{2m\pi x}{R}\cos\frac{4m\pi x}{R}\cos\frac{2n\pi y}{R}$$

$$\mp\frac{1}{2}\cos\frac{3m\pi x}{R}\cos\frac{4m\pi x}{R}\cos\frac{n\pi y}{R} \mp\frac{9}{2}\cos\frac{m\pi x}{R}\cos\frac{4m\pi x}{R}\cos\frac{3n\pi y}{R} \mp \cos\frac{4m\pi x}{R}\cos\frac{4n\pi y}{R}\Big\}$$

$$+ 3\cos\frac{m\pi x}{R}\cos\frac{n\pi y}{R} + 4\cos\frac{2m\pi x}{R} + 3\cos\frac{2m\pi x}{R}\cos\frac{2n\pi y}{R} + \frac{9}{2}\cos\frac{3m\pi x}{R}\cos\frac{n\pi y}{R}$$

$$+\frac{1}{2}\cos\frac{m\pi x}{R}\cos\frac{3n\pi y}{R} + \cos\frac{4m\pi x}{R}\Big]$$

$$\Delta\Delta F = \left(\frac{m}{R}\right)^2 E \frac{f_1}{8}\Big[\left(\frac{f_1}{8} m^2\right)\Big\{ -\frac{9}{2}\cos\frac{2mx}{R} - \frac{9}{2}\cos\frac{2my}{2} - \frac{9}{2}\cos\frac{4mx}{R} - \frac{9}{2}\cos\frac{4my}{R} - \frac{9}{8}\cos\frac{6mx}{2} - \frac{9}{8}\cos\frac{2my}{R}$$

$$-\frac{9}{8}\cos\frac{2mx}{R} - \frac{9}{8}\cos\frac{6my}{R} - 9\cos\frac{3mx}{R}\cos\frac{my}{R} - 9\cos\frac{mx}{R}\cos\frac{3my}{R} - \frac{3}{2}\cos\frac{2mx}{R} - 6\cos\frac{4mx}{R}$$

$$-6\cos\frac{2mx}{R}\cos\frac{2my}{R} - \frac{3}{2}\cos\frac{4mx}{R}\cos\frac{2my}{R} - \frac{3}{2}\cos\frac{2my}{R} - 6\cos\frac{4my}{R} - 6\cos\frac{2mx}{R}\cos\frac{2my}{R}$$

$$-\frac{3}{2}\cos\frac{2mx}{R}\cos\frac{4my}{R} - \frac{3}{2}\cos\frac{mx}{R}\cos\frac{my}{R} - 6\cos\frac{5mx}{R}\cos\frac{my}{R} - 6\cos\frac{mx}{R}\cos\frac{3my}{R} - \frac{3}{2}\cos\frac{5mx}{2}\cos\frac{3my}{R}$$

$$-\frac{3}{2}\cos\frac{mx}{R}\cos\frac{my}{R} - 6\cos\frac{mx}{R}\cos\frac{5my}{R} - 6\cos\frac{3mx}{R}\cos\frac{my}{R} - \frac{3}{2}\cos\frac{3mx}{R}\cos\frac{5my}{R}$$

$$-\frac{9}{2}\cos\frac{4mx}{R}\cos\frac{2my}{R} - \frac{9}{2}\cos\frac{2mx}{R}\cos\frac{4my}{R} - 6\cos\frac{mx}{R}\cos\frac{my}{R} - 6\cos\frac{mx}{R}\cos\frac{3my}{R}$$

$$-16\cos\frac{2mx}{R}\cos\frac{2my}{R} - 6\cos\frac{2mx}{R} - 6\cos\frac{2mx}{R}\cos\frac{4my}{R} - 9\cos\frac{3mx}{R}\cos\frac{my}{R} - 9\cos\frac{3mx}{R}\cos\frac{3my}{R}$$

$$-\cos\frac{mx}{R}\cos\frac{my}{R} - \cos\frac{mx}{R}\cos\frac{5my}{R} - 4\cos\frac{4mx}{R}\cos\frac{2my}{R} - \frac{3}{2}\cos\frac{mx}{R}\cos\frac{3my}{R} - \frac{3}{2}\cos\frac{mx}{R}\cos\frac{5my}{R}$$

$$-4\cos\frac{2mx}{R}\cos\frac{4my}{R} - \frac{3}{2}\cos\frac{2mx}{R}\cos\frac{2my}{R} - \frac{3}{2}\cos\frac{2mx}{2}\cos\frac{6my}{R} - \frac{9}{4}\cos\frac{3mx}{R}\cos\frac{3my}{R}$$

$$-\frac{9}{4}\cos\frac{3mx}{R}\cos\frac{5my}{R} - \frac{1}{4}\cos\frac{mx}{R}\cos\frac{my}{R} - \frac{1}{4}\cos\frac{mx}{R}\cos\frac{7my}{R} - \cos\frac{4mx}{R}\cos\frac{4my}{R}$$

$$-6\cos\frac{mx}{R}\cos\frac{my}{R} - 6\cos\frac{3mx}{R}\cos\frac{my}{R} - 16\cos\frac{2mx}{R}\cos\frac{2my}{R} - 6\cos\frac{2my}{R} - 6\cos\frac{4mx}{R}\cos\frac{2my}{R}$$

$$-\cos\frac{mx}{R}\cos\frac{my}{R} - \cos\frac{5mx}{R}\cos\frac{my}{R} - 9\cos\frac{mx}{R}\cos\frac{3my}{R} - 9\cos\frac{3mx}{R}\cos\frac{3my}{R} - 4\cos\frac{2mx}{R}\cos\frac{4my}{R}$$

$$-\frac{3}{2}\cos\frac{3mx}{R}\cos\frac{my}{R} - \frac{3}{2}\cos\frac{5mx}{R}\cos\frac{my}{R} - 4\cos\frac{4mx}{R}\cos\frac{2my}{R} - \frac{3}{2}\cos\frac{2mx}{R}\cos\frac{2my}{R} - \frac{3}{2}\cos\frac{6mx}{R}\cos\frac{2my}{R}$$

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$$(1) = \frac{9}{2}\left[\left(\cos\frac{\pi x}{R} - \cos\frac{3\pi x}{R}\right)\left(\cos\frac{\pi y}{R} - \cos\frac{3\pi y}{R}\right) - \left(\cos\frac{\pi x}{R} + \cos\frac{3\pi x}{R}\right)\left(\cos\frac{\pi y}{R} + \cos\frac{3\pi y}{R}\right)\right]$$

$$= \frac{9}{2}\left[-2\cos\frac{3\pi x}{R}\cos\frac{\pi y}{R} - 2\cos\frac{\pi x}{R}\cos\frac{3\pi y}{R}\right] = -9\left[\cos\frac{3\pi x}{R}\cos\frac{\pi y}{R} + \cos\frac{\pi x}{R}\cos\frac{3\pi y}{R}\right]$$

$$(2) = \frac{3}{4}\left[3\left(\cos\frac{2\pi x}{R} - \cos\frac{4\pi x}{R}\right)\left(1 - \cos\frac{2\pi y}{R}\right) - 5\left(\cos\frac{2\pi x}{R} + \cos\frac{4\pi x}{R}\right)\left(1 + \cos\frac{2\pi y}{R}\right)\right]$$

$$= \frac{3}{4}\left[-2\cos\frac{2\pi x}{R} - 8\cos\frac{4\pi x}{R} - 8\cos\frac{2\pi x}{R}\cos\frac{2\pi y}{R} - 2\cos\frac{4\pi x}{R}\cos\frac{2\pi y}{R}\right]$$

$$= -\frac{3}{2}\left[\cos\frac{2\pi x}{R} + 4\cos\frac{4\pi x}{R} + 4\cos\frac{2\pi x}{R}\cos\frac{2\pi y}{R} + \cos\frac{4\pi x}{R}\cos\frac{2\pi y}{R}\right]$$

$$(3) = -\frac{3}{2}\left[\cos\frac{2\pi y}{R} + 4\cos\frac{4\pi y}{R} + 4\cos\frac{2\pi x}{R}\cos\frac{2\pi y}{R} + \cos\frac{2\pi x}{R}\cos\frac{4\pi y}{R}\right]$$

$$(4) = \frac{3}{4}\left[3\left(\cos\frac{\pi x}{R} - \cos\frac{5\pi x}{R}\right)\left(\cos\frac{\pi y}{R} - \cos\frac{3\pi y}{R}\right) - 5\left(\cos\frac{\pi x}{R} + \cos\frac{5\pi x}{R}\right)\left(\cos\frac{\pi y}{R} + \cos\frac{3\pi y}{R}\right)\right]$$

$$= \frac{3}{4}\left[-2\cos\frac{\pi x}{R}\cos\frac{\pi y}{R} - 8\cos\frac{5\pi x}{R}\cos\frac{\pi y}{R} - 8\cos\frac{\pi x}{R}\cos\frac{3\pi y}{R} - 2\cos\frac{5\pi x}{R}\cos\frac{3\pi y}{R}\right]$$

$$= -\frac{3}{2}\left[\cos\frac{\pi x}{R}\cos\frac{\pi y}{R} + 4\cos\frac{5\pi x}{R}\cos\frac{\pi y}{R} + 4\cos\frac{\pi x}{R}\cos\frac{3\pi y}{R} + \cos\frac{5\pi x}{R}\cos\frac{3\pi y}{R}\right]$$

$$(5) = -\frac{3}{2}\left[\cos\frac{\pi x}{R}\cos\frac{\pi y}{R} + 4\cos\frac{\pi x}{R}\cos\frac{5\pi y}{R} + 4\cos\frac{3\pi x}{R}\cos\frac{\pi y}{R} + \cos\frac{3\pi x}{R}\cos\frac{5\pi y}{R}\right]$$

618

$$(6) = \frac{q}{4}\left[\left(\cos\frac{2\pi x}{R} - \cos\frac{4\pi x}{R}\right)\left(\cos\frac{2\pi y}{R} - \cos\frac{4\pi y}{R}\right) - \left(\cos\frac{2\pi x}{R} + \cos\frac{4\pi x}{R}\right)\left(\cos\frac{2\pi y}{R} + \cos\frac{4\pi y}{R}\right)\right]$$

$$= -\frac{q}{2}\left[\cos\frac{4\pi x}{R}\cos\frac{2\pi y}{R} + \cos\frac{2\pi x}{R}\cos\frac{4\pi y}{R}\right]$$

$$-\frac{1}{4}\cos\frac{\pi x}{R}\cos\frac{\pi y}{R}-\frac{1}{4}\cos\frac{7\pi x}{R}\cos\frac{\pi y}{R}-\frac{9}{4}\cos\frac{3\pi x}{R}\cos\frac{3\pi y}{R}-\frac{9}{4}\cos\frac{5\pi x}{R}\cos\frac{3\pi y}{R}$$

$$-\cos\frac{4\pi x}{R}\cos\frac{4\pi y}{R}\Big\}+3\cos\frac{\pi x}{R}\cos\frac{\pi y}{R}+4\cos\frac{2\pi x}{R}+3\cos\frac{2\pi x}{R}\cos\frac{2\pi y}{R}+\frac{9}{2}\cos\frac{3\pi x}{R}\cos\frac{\pi y}{R}$$

$$+\frac{1}{2}\cos\frac{\pi x}{R}\cos\frac{3\pi y}{R}+\cos\frac{4\pi x}{R}\Big]$$

619

$$\Delta\Delta F = \left(\frac{m}{R}\right)^2 E\,\frac{f_1}{8}$$

$$\Big[\ \cos\frac{2mx}{R}\Big\{\left(-\frac{f_1 m^2}{8}\right)\Big(\overbrace{\frac{9}{2}+\frac{9}{8}+\frac{3}{2}+6}^{\frac{105}{8}}\Big)+4\Big\}$$

$$+\cos\frac{2my}{R}\Big\{\left(-\frac{f_1 m^2}{8}\right)\Big(\overbrace{\frac{9}{2}+\frac{9}{8}+\frac{3}{2}+6}^{\frac{105}{8}}\Big)\Big\}$$

$$+\cos\frac{4mx}{R}\Big\{\left(-\frac{f_1 m^2}{8}\right)\Big(\overbrace{\frac{9}{2}+6}^{\frac{21}{2}}\Big)+1\Big\}$$

$$+\cos\frac{4my}{R}\Big\{\left(-\frac{f_1 m^2}{8}\right)\Big(\overbrace{\frac{9}{2}+6}^{\frac{21}{2}}\Big)\Big\}$$

$$+\cos\frac{6mx}{R}\Big\{\left(-\frac{f_1 m^2}{8}\right)\left(\frac{9}{8}\right)\Big\}$$

$$+\cos\frac{6my}{R}\Big\{\left(-\frac{f_1 m^2}{8}\right)\left(\frac{9}{8}\right)\Big\}$$

$$+\cos\frac{mx}{R}\cos\frac{my}{R}\Big\{\left(-\frac{f_1 m^2}{8}\right)\Big(\overbrace{\frac{3}{2}+\frac{3}{2}+6+1+\frac{1}{4}+6+1+\frac{1}{4}}^{\frac{35}{2}}\Big)+3\Big\}$$

$$+\cos\frac{2mx}{R}\cos\frac{2my}{R}\Big\{\left(-\frac{f_1 m^2}{8}\right)\Big(\overbrace{6+6+16+\frac{3}{2}+16+\frac{3}{2}}^{47}\Big)+3\Big\}$$

$$+\cos\frac{mx}{R}\cos\frac{3my}{R}\Big\{\left(-\frac{f_1 m^2}{8}\right)\Big(\overbrace{9+6+6+\frac{3}{2}+9}^{\frac{63}{2}}\Big)+\frac{1}{2}\Big\}$$

$$+\cos\frac{3mx}{R}\cos\frac{my}{R}\Big\{\left(-\frac{f_1 m^2}{8}\right)\Big(\overbrace{9+6+9+6+\frac{3}{2}}^{\frac{63}{2}}\Big)+\frac{9}{2}\Big\}$$

$$+\cos\frac{3mx}{R}\cos\frac{3my}{R}\Big\{\left(-\frac{f_1 m^2}{8}\right)\Big(\overbrace{9+\frac{9}{4}+9+\frac{9}{4}}^{\frac{45}{2}}\Big)\Big\}$$

$$+\cos\frac{4mx}{R}\cos\frac{2my}{R}\Big\{\left(-\frac{f_1 m^2}{8}\right)\Big(\overbrace{\frac{3}{2}+\frac{9}{2}+4+6+4}^{20}\Big)\Big\}$$

$$+\cos\frac{2mx}{R}\cos\frac{4my}{R}\Big\{\left(-\frac{f_1 m^2}{8}\right)\Big(\overbrace{\frac{3}{2}+\frac{9}{2}+6+4+4}^{20}\Big)\Big\}$$

$$+\cos\frac{5mx}{R}\cos\frac{my}{R}\left\{\left(-\frac{f_1m^2}{8}\right)\left(\overbrace{6+1+\frac{3}{2}}^{\frac{17}{2}}\right)\right\}$$

$$+\cos\frac{mx}{R}\cos\frac{5my}{R}\left\{\left(-\frac{f_1m^2}{8}\right)\left(\overbrace{6+1+\frac{3}{2}}^{\frac{17}{2}}\right)\right\}$$

$$+\cos\frac{3mx}{R}\cos\frac{5my}{R}\left\{\left(-\frac{f_1m^2}{8}\right)\left(\overbrace{\frac{3}{2}+\frac{9}{4}}^{\frac{15}{4}}\right)\right\}$$

$$+\cos\frac{5mx}{R}\cos\frac{3my}{R}\left\{\left(-\frac{f_1m^2}{8}\right)\left(\overbrace{\frac{3}{2}+\frac{9}{4}}^{\frac{15}{4}}\right)\right\}$$

$$+\cos\frac{6mx}{R}\cos\frac{2my}{R}\left\{\left(-\frac{f_1m^2}{8}\right)\left(\frac{3}{2}\right)\right\}$$

$$+\cos\frac{2mx}{R}\cos\frac{6my}{R}\left\{\left(-\frac{f_1m^2}{8}\right)\left(\frac{3}{2}\right)\right\}$$

$$+\cos\frac{mx}{R}\cos\frac{7my}{R}\left\{\left(-\frac{f_1m^2}{8}\right)\left(\frac{1}{4}\right)\right\}$$

$$+\cos\frac{7mx}{R}\cos\frac{my}{R}\left\{\left(-\frac{f_1m^2}{8}\right)\left(\frac{1}{4}\right)\right\}$$

$$+\cos\frac{4mx}{R}\cos\frac{4my}{R}\left\{\left(-\frac{f_1m^2}{8}\right)(1+1)\right\}$$

$$F=\left(\frac{R}{m}\right)^2 E\,\frac{f_1}{8}\Big[-\frac{1}{16}\left(\frac{105}{64}f_1m^2-4\right)\cos\frac{2mx}{R}-\frac{1}{16}\,\frac{105}{64}f_1m^2\cos\frac{2my}{R}-\frac{1}{256}\left(\frac{21}{16}f_1m^2-1\right)\cos\frac{4mx}{R}$$

$$-\frac{1}{256}\,\frac{21}{16}f_1m^2\cos\frac{4my}{R}-\frac{1}{1296}\,\frac{9}{64}f_1m^2\cos\frac{6mx}{R}-\frac{1}{1296}\,\frac{9}{64}f_1m^2\cos\frac{6my}{R}-\frac{1}{4}\left(\frac{35}{16}f_1m^2-3\right)\cos\frac{mx}{R}\cos\frac{my}{R}$$

$$-\frac{1}{64}\left(\frac{47}{8}f_1m^2-3\right)\cos\frac{2mx}{R}\cos\frac{2my}{R}-\frac{1}{100}\left(\frac{63}{16}f_1m^2-\frac{1}{2}\right)\cos\frac{mx}{R}\cos\frac{3my}{R}-\frac{1}{100}\left(\frac{63}{16}f_1m^2-\frac{1}{2}\right)\cos\frac{3mx}{R}\cos\frac{my}{R}$$

$$-\frac{1}{324}\,\frac{45}{16}f_1m^2\cos\frac{3mx}{R}\cos\frac{3my}{R}-\frac{1}{400}\,\frac{5}{2}f_1m^2\cos\frac{4mx}{R}\cos\frac{2my}{R}-\frac{1}{400}\,\frac{5}{2}f_1m^2\cos\frac{2mx}{R}\cos\frac{4my}{R}$$

$$-\frac{1}{676}\,\frac{17}{16}f_1m^2\cos\frac{5mx}{R}\cos\frac{my}{R}-\frac{1}{676}\,\frac{17}{16}f_1m^2\cos\frac{mx}{R}\cos\frac{5my}{R}-\frac{1}{1156}\,\frac{15}{32}f_1m^2\cos\frac{3mx}{R}\cos\frac{5my}{R}$$

$$-\frac{1}{1156}\,\frac{15}{32}f_1m^2\cos\frac{5mx}{R}\cos\frac{3my}{R}-\frac{1}{1600}\,\frac{3}{16}f_1m^2\cos\frac{6mx}{R}\cos\frac{2my}{R}-\frac{1}{1600}\,\frac{3}{16}f_1m^2\cos\frac{2mx}{R}\cos\frac{6my}{R}$$

$$-\frac{1}{2500}\,\frac{1}{32}f_1m^2\cos\frac{mx}{R}\cos\frac{7my}{R}-\frac{1}{2500}\,\frac{1}{32}f_1m^2\cos\frac{7mx}{R}\cos\frac{my}{R}-\frac{1}{1024}\,\frac{1}{4}f_1m^2\cos\frac{4mx}{R}\cos\frac{4my}{R}\Big]$$

$$-\frac{\sigma}{2}y^2+\frac{\lambda}{2}x^2$$

$$\sigma_x = E\frac{f_1}{8}\Big[+\frac{1}{4}\frac{105}{64}f_1m^2\cos\frac{2my}{R}+\frac{1}{16}\frac{21}{16}f_1m^2\cos\frac{4my}{R}+\frac{1}{36}\frac{9}{64}f_1m^2\cos\frac{6my}{R}$$

$$+\frac{1}{4}\left(\frac{35}{16}f_1m^2-3\right)\cos\frac{mx}{R}\cos\frac{my}{R}+\frac{1}{16}\left(\frac{47}{8}f_1m^2-3\right)\cos\frac{2mx}{R}\cos\frac{2my}{R}+\frac{9}{100}\left(\frac{63}{16}f_1m^2-\frac{1}{2}\right)\cos\frac{mx}{R}\cos\frac{3my}{R}$$

$$+\frac{1}{100}\left(\frac{63}{16}f_1m^2-\frac{1}{2}\right)\cos\frac{3mx}{R}\cos\frac{my}{R}+\frac{9}{324}\frac{45}{16}f_1m^2\cos\frac{3mx}{R}\cos\frac{3my}{R}+\frac{4}{400}\frac{5}{2}f_1m^2\cos\frac{4mx}{R}\cos\frac{2my}{R}$$

$$+\frac{16}{400}\frac{5}{2}f_1m^2\cos\frac{2mx}{R}\cos\frac{4my}{R}+\frac{1}{676}\frac{17}{16}f_1m^2\cos\frac{5mx}{R}\cos\frac{my}{R}+\frac{25}{676}\frac{17}{16}f_1m^2\cos\frac{mx}{R}\cos\frac{5my}{R}$$

$$+\frac{25}{1156}\frac{15}{32}f_1m^2\cos\frac{3mx}{R}\cos\frac{5my}{R}+\frac{9}{1156}\frac{15}{32}f_1m^2\cos\frac{5mx}{R}\cos\frac{3my}{R}+\frac{4}{1600}\frac{3}{16}f_1m^2\cos\frac{6mx}{R}\cos\frac{2my}{R}$$

$$+\frac{36}{1600}\frac{3}{16}f_1m^2\cos\frac{2mx}{R}\cos\frac{6my}{R}+\frac{49}{2500}\frac{1}{32}f_1m^2\cos\frac{mx}{R}\cos\frac{7my}{R}+\frac{1}{2500}\frac{1}{32}f_1m^2\cos\frac{7mx}{R}\cos\frac{my}{R}$$

$$+\frac{16}{1024}\frac{1}{4}f_1m^2\cos\frac{4mx}{R}\cos\frac{4my}{R}\Big]-\sigma$$

$$-\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2=-\frac{1}{2}m^2\left(\frac{f_1}{8}\right)^2\Big[\frac{9}{4}\left(1+\cos\frac{2mx}{R}\right)\left(1-\cos\frac{2my}{R}\right)+2-2\cos\frac{4my}{R}+\frac{9}{16}\left(1+\cos\frac{4mx}{R}\right)\left(1-\cos\frac{4my}{R}\right)$$

$$+\frac{1}{16}\left(1+\cos\frac{6mx}{R}\right)\left(1-\cos\frac{2my}{R}\right)+\frac{9}{16}\left(1+\cos\frac{2mx}{R}\right)\left(1-\cos\frac{6my}{R}\right)+\frac{1}{32}-\frac{1}{32}\cos\frac{8my}{R}$$

$$+6\cos\frac{mx}{R}\left(\cos\frac{my}{R}-\cos\frac{3my}{R}\right)+\frac{9}{4}\left(\cos\frac{mx}{R}+\cos\frac{3mx}{R}\right)\left(\cos\frac{my}{R}-\cos\frac{3my}{R}\right)+\frac{3}{4}\left(\cos\frac{2mx}{R}+\cos\frac{4mx}{R}\right)\left(1-\cos\frac{2my}{R}\right)$$

$$+\frac{9}{4}\left(1+\cos\frac{2mx}{R}\right)\left(\cos\frac{2my}{R}-\cos\frac{4my}{R}\right)+\frac{3}{4}\cos\frac{mx}{R}\left(\cos\frac{3my}{R}-\cos\frac{5my}{R}\right)+\text{------}\Big]$$

623

$$-\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 = -\frac{1}{2}m^2\left(\frac{f_1}{8}\right)^2\left[\frac{9}{4} + 2 + \frac{9}{16} + \frac{1}{16} + \frac{9}{16} + \frac{1}{32} + \cdots\right]$$

$$= -\frac{1}{2}m^2\left(\frac{f_1}{8}\right)^2 \frac{175}{32} + \cdots$$

$$\left(\frac{\lambda}{E} + \nu\frac{\sigma}{E}\right) - \frac{175}{4096}f_1(f_1 m^2) + \left(f_0 + \frac{9}{64}f_1\right) = 0$$

$$\boxed{\frac{\lambda}{E} = \frac{175}{4096}f_1(f_1 m^2) - \left(f_0 + \frac{9}{64}f_1\right) - \nu\frac{\sigma}{E}}$$

$$p_1 = -8\frac{\sigma}{E}\left[\left(\frac{\sigma}{E} + \nu\frac{\lambda}{E}\right) + \frac{175}{4096}f_1(f_1 m^2)\right]$$

$$\boxed{p_1 = -4\frac{\sigma}{E}\left[2(1-\nu^2)\frac{\sigma}{E} + \frac{175}{2048}(1+\nu)f_1(f_1 m^2) - 2\nu\left(f_0 + \frac{9}{64}f_1\right)\right]}$$

$$4\left[\left(\frac{\sigma}{E}\right)^2 + \left(\frac{\lambda}{E}\right)^2 + 2\nu\frac{\sigma}{E}\frac{\lambda}{E}\right]$$

$$= 4\left[\left(\frac{\sigma}{E}\right)^2 + \frac{175^2}{4096^2}f_1^2(f_1 m^2)^2 + \left(f_0 + \frac{9}{64}f_1\right)^2 + \nu^2\left(\frac{\sigma}{E}\right)^2 - \frac{175}{2048}f_1(f_1 m^2)\left(f_0 + \frac{9}{64}f_1\right)\right.$$

$$\left. - \cancel{\frac{175}{2048}\nu\frac{\sigma}{E}f_1(f_1 m^2)} + \cancel{2\nu\frac{\sigma}{E}\left(f_0 + \frac{9}{64}f_1\right)} + \cancel{\nu\frac{\sigma}{E}\frac{175}{2048}f_1(f_1 m^2)} - \cancel{2\nu\frac{\sigma}{E}\left(f_0 + \frac{9}{64}f_1\right)} - 2\nu^2\left(\frac{\sigma}{E}\right)^2\right]$$

624

$$\rho_1 + 4\left[\left(\frac{\sigma}{E}\right)^2 + \left(\frac{\lambda}{E}\right)^2 + 2\nu \frac{\sigma}{E}\frac{\lambda}{E}\right] = K$$

$$= -4\left[(1-\nu^2)\left(\frac{\sigma}{E}\right)^2 + \frac{175}{2048}(1+\nu) f_1 (f_1 m^2)\frac{\sigma}{E} - 2\nu\left(f_0 + \frac{9}{64} f_1\right)\frac{\sigma}{E} - \frac{175^2}{4096^2} f_1^2 (f_1 m^2)^2 - \left(f_0 + \frac{9}{64} f_1\right)^2 \right.$$

$$\left. + \frac{175}{2048} f_1 (f_1 m^2)\left(f_0 + \frac{9}{64} f_1\right)\right]$$

$$\frac{\partial K}{\partial f_0} = 0, \qquad \boxed{-2\nu\frac{\sigma}{E} - 2\left(f_0 + \frac{9}{64} f_1\right) + \frac{175}{2048} f_1 (f_1 m^2) = 0}$$

$$K = -4\left[(1-\nu^2)\left(\frac{\sigma}{E}\right)^2 + \frac{175}{2048}(1+\nu) f_1 (f_1 m^2)\frac{\sigma}{E} - \frac{175^2}{4096^2} f_1^2 (f_1 m^2)^2 + \left(f_0 + \frac{9}{64} f_1\right)^2\right]$$

$$\boxed{K = -4\left(\frac{\sigma}{E}\right)^2 - \frac{175}{512}\left(\frac{\sigma}{E}\right) f_1 (f_1 m^2)}$$

625

$$\delta_2 - \text{Constant Part} = \frac{f_1^2}{64}\Big[(f_1 m^2)^2\Big(\frac{1}{8}\frac{105^2}{64^2} + \frac{1}{128}\frac{21^2}{16^2} + \frac{1}{648}\frac{81}{64^2} + \frac{1}{8}\frac{105^2}{64^2} + \frac{1}{128}\frac{21^2}{16^2} + \frac{1}{648}\frac{81}{64^2}$$

$$+ \frac{1}{4}\frac{35^2}{16^2} + \frac{1}{64}\frac{47^2}{64} + \frac{1}{100}\frac{63^2}{16^2} + \frac{1}{100}\frac{63^2}{16^2} + \frac{1}{324}\frac{45^2}{16^2} + \frac{1}{400}\frac{25}{4} + \frac{1}{400}\frac{25}{4} + \frac{1}{676}\frac{17^2}{16^2} + \frac{1}{676}\frac{17^2}{16^2}$$

$$+ \frac{2}{1156}\frac{225}{32^2} + \frac{2}{1600}\frac{9}{16^2} + \frac{2}{2500}\frac{1}{32^2} + \frac{1}{1024}\frac{1}{16}\Big) - (f_1 m^2)\Big(\frac{1}{8}\frac{105}{8} + \frac{1}{128}\frac{21}{8} + \frac{1}{4}\frac{105}{8} + \frac{1}{64}\frac{141}{4}$$

$$+ \frac{1}{100}\frac{63}{16} + \frac{1}{100}\frac{63}{16}\Big) + \Big(2 + \frac{1}{128} + \frac{9}{4} + \frac{9}{64} + \frac{1}{200}\Big)\Big]$$

$$= \frac{f_1^2}{64}\Big[(f_1 m^2)^2\Big(\frac{1}{4}\frac{105^2}{64^2} + \frac{1}{64}\frac{21^2}{16^2} + \frac{1}{324}\frac{81}{64^2} + \frac{1}{4}\frac{35^2}{16^2} + \frac{1}{64}\frac{47^2}{64} + \frac{1}{50}\frac{63^2}{16^2} + \frac{1}{324}\frac{45^2}{16^2}$$

$$+ \frac{1}{32} + \frac{1}{338}\frac{17^2}{16^2} + \frac{1}{578}\frac{225}{32^2} + \frac{1}{800}\frac{9}{16^2} + \frac{1}{1250}\frac{1}{32^2} + \frac{1}{1024}\frac{1}{16}\Big)$$

$$- \frac{1}{64}(f_1 m^2)\Big(105 + \frac{21}{16} + 210 + \frac{141}{4} + \frac{126}{25}\Big) + \Big(2 + \frac{1}{128} + \frac{9}{4} + \frac{9}{64} + \frac{1}{200}\Big)\Big]$$

$$= \frac{f_1^2}{64}\Big[\frac{1}{64^2}(f_1 m^2)^2\big(2756.25 + 110.25 + 0.25 + 4900 + 2209 + 1270.08 + 100 + 128 + 13.680$$

$$+ 1.557 + 0.18 + 0.003 + 0.25\big) - \frac{1}{64}(f_1 m^2)\big(315 + 1.3125 + 35.25 + 5.04\big)$$

$$+ \big(2 + 2.3984 + 0.005\big)$$

$$= \frac{f_1^2}{64}\Big[2.80505\,(f_1 m^2)^2 - 5.57191\,(f_1 m^2) + 4.4034\Big]$$

626

$$P_3 = \frac{1}{12(1-\nu^2)}\left(\frac{t}{R}\right)^2\left(\frac{f_1}{8}\right)^2 m^4\left[36 + 32 + 32 + 36 + 25 + 25 + 2\right]$$

$$= \frac{1}{12(1-\nu^2)}\,\frac{1}{\cancel{64}_{16}} \times \overset{47}{\cancel{188}}\quad m^4\left(\frac{t}{R}\right)^2 f_1^2 = \frac{47}{192(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4 f_1^2$$

Total potential

$$-4\left(\frac{\sigma}{E}\right)^2 - \frac{175}{512}\left(\frac{\sigma}{E}\right) f_1^2 m^2 + \frac{f_1^2}{64}\left[2.80505\,(f_1 m^2)^2 - 5.57191\,(f_1 m^2) + 4.4034\right] + \frac{47}{192(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4 f_1^2$$

$$\therefore\quad \frac{175}{256}\left(\frac{\sigma}{E}\right) m^2 = \frac{1}{32}\left[5.61010\,(f_1 m^2)^2 - 8.35787\,(f_1 m^2) + 4.4034\right] + \frac{47}{96(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4$$

$$\frac{\sigma}{E} = \left[0.25646\, f_1^2 + 0.78702\left(\frac{t}{R}\right)^2\right] m^2 - 0.38207\, f_1 + 0.20130\,\frac{1}{m^2}$$

$$\frac{\sigma}{E}\frac{R}{t} = \left[0.20650\left(\frac{\delta}{t}\right)^2 + 0.63370\right]^{\frac{1}{2}} - 0.38207\left(\frac{\delta}{t}\right)$$

$$\left\{(0.20650)^2 - 0.38207^2 \times 0.20650\right\}\left(\frac{\delta}{t}\right)^2 = 0.38207^2 \times 0.63370$$

$$\left(\frac{\delta}{t}\right)^2 = \frac{0.38207^2 \times 0.63370}{0.20650 \times 0.06052} = 10.471 \times \frac{0.14598}{0.20650}$$

628

$$\frac{w}{R} = \left(f_0 + \frac{1}{4} f_1\right) + \frac{1}{2} f_1 \left[\cos\frac{mx}{R} \cos\frac{my}{R} + \frac{1}{4}\cos\frac{2mx}{R} + \frac{1}{4}\cos\frac{2my}{R} \right]$$

$$+ \frac{1}{2} f_2 \left[\cos\frac{mx}{R} + \cos\frac{my}{R} \right]$$

$$\left(\frac{\partial w}{\partial y}\right) = -m \left[\frac{1}{2} f_1 \left\{ \cos\frac{mx}{R} \sin\frac{my}{R} + \frac{1}{2}\sin\frac{2my}{R} \right\} + \frac{1}{2} f_2 \sin\frac{my}{R} \right]$$

$$\frac{1}{R}\frac{\partial^2 w}{\partial x^2} = -\left(\frac{m}{R}\right)^2 \left\{ \frac{1}{2} f_1 \left[\cos\frac{mx}{R}\cos\frac{my}{R} + \cos\frac{2mx}{R} \right] + \frac{1}{2} f_2 \cos\frac{mx}{R} \right\}$$

$$\frac{1}{R}\frac{\partial^2 w}{\partial y^2} = -\left(\frac{m}{R}\right)^2 \left\{ \frac{1}{2} f_1 \left[\cos\frac{mx}{R}\cos\frac{my}{R} + \cos\frac{2my}{R} \right] + \frac{1}{2} f_2 \cos\frac{my}{R} \right\}$$

$$\frac{1}{R}\frac{\partial^2 w}{\partial x \partial y} = \left(\frac{m}{R}\right)^2 \left\{ \frac{1}{2} f_1 \sin\frac{mx}{R} \sin\frac{my}{R} \right\}$$

$$\Delta\Delta F = E\left(\frac{m}{R}\right)^2 \left[\left(\frac{1}{2} f_1 m\right)^2 \left\{ -\frac{1}{2}\cos\frac{2mx}{R} - \frac{1}{2}\cos\frac{2my}{R} \right.\right.$$

$$- \frac{1}{2}\left(\cos\frac{mx}{R} + \cos\frac{3mx}{R}\right)\cos\frac{my}{R} - \frac{1}{2}\cos\frac{mx}{R}\left(\cos\frac{my}{R} + \cos\frac{3my}{R}\right) - \cos\frac{2mx}{R}\cos\frac{2my}{R} \Big\}$$

$$+ \frac{1}{4} f_1 f_2 m^2 \left\{ -\frac{1}{2}\cos\frac{mx}{R}\left(1 + \cos\frac{2my}{R}\right) - \frac{1}{2}\left(1 + \cos\frac{2mx}{R}\right)\cos\frac{my}{R} \right.$$

$$\left. - \cos\frac{2mx}{R}\cos\frac{my}{R} - \cos\frac{mx}{R}\cos\frac{2my}{R} \right\} - \frac{1}{4} f_2^2 m^2 \cos\frac{mx}{R}\cos\frac{my}{R}$$

$$\left. + \frac{1}{2} f_1 \left\{ \cos\frac{mx}{R}\cos\frac{my}{R} + \cos\frac{2mx}{R} \right\} + \frac{1}{2} f_2 \cos\frac{mx}{R} \right]$$

$$\Delta\Delta F = E\left(\frac{m}{R}\right)^2\Big[\frac{1}{2}f_2\left(1-\frac{1}{4}f_1 m^2\right)\cos\frac{mx}{R} - \frac{1}{8}f_1 f_2 m^2\cos\frac{my}{R} + \frac{1}{2}f_1\left(1-\frac{1}{4}f_1 m^2\right)\cos\frac{2mx}{R}$$

$$-\frac{1}{8}f_1^2 m^2\cos\frac{2my}{R} + \left(\frac{1}{2}f_1 - \frac{1}{4}f_1^2 m^2 - \frac{1}{4}f_2^2 m^2\right)\cos\frac{mx}{R}\cos\frac{my}{R} - \frac{3}{8}f_1 f_2 m^2\cos\frac{2mx}{R}\cos\frac{my}{R}$$

$$-\frac{3}{8}f_1 f_2 m^2\cos\frac{mx}{R}\cos\frac{2my}{R} - \frac{1}{8}f_1^2 m^2\cos\frac{3mx}{R}\cos\frac{my}{R} - \frac{1}{8}f_1^2 m^2\cos\frac{mx}{R}\cos\frac{3my}{R} - \frac{1}{4}f_1^2 m^2\cos\frac{2mx}{R}\cos\frac{2my}{R}\Big]$$

$$F = E\left(\frac{R}{m}\right)^2\frac{1}{2}\Big[f_2\left(1-\frac{1}{4}f_1 m^2\right)\cos\frac{mx}{R} - \frac{1}{4}f_1 f_2 m^2\cos\frac{my}{R} + \frac{1}{16}f_1\left(1-\frac{1}{4}f_1 m^2\right)\cos\frac{2mx}{R}$$

$$-\frac{1}{16}\frac{1}{4}f_1^2 m^2\cos\frac{2my}{R} + \frac{1}{4}\left(f_1 - \frac{1}{2}f_1^2 m^2 - \frac{1}{2}f_2^2 m^2\right)\cos\frac{mx}{R}\cos\frac{my}{R} - \frac{1}{25}\frac{3}{4}f_1 f_2 m^2\cos\frac{2mx}{R}\cos\frac{my}{R}$$

$$-\frac{1}{25}\frac{3}{4}f_1 f_2 m^2\cos\frac{mx}{R}\cos\frac{2my}{R} - \frac{1}{100}\frac{1}{4}f_1^2 m^2\cos\frac{3mx}{R}\cos\frac{my}{R} - \frac{1}{100}\frac{1}{4}f_1^2 m^2\cos\frac{mx}{R}\cos\frac{3my}{R} - \frac{1}{64}\frac{1}{4}f_1^2 m^2\cos\frac{2mx}{R}\cos\frac{2my}{R}\Big]$$

$$\sigma_x + \sigma_y = (-\sigma+\lambda) + E\frac{1}{2}\Big[f_2\left(\frac{1}{4}f_1 m^2 - 1\right)\cos\frac{mx}{R} + \frac{1}{4}f_1 f_2 m^2\cos\frac{my}{R} + \frac{1}{4}f_1\left(\frac{1}{4}f_1 m^2 - 1\right)\cos\frac{2mx}{R}$$

$$+\frac{1}{16}f_1^2 m^2\cos\frac{2my}{R} + \frac{1}{2}\left(\frac{1}{2}f_1^2 m^2 + \frac{1}{2}f_2^2 m^2 - f_1\right)\cos\frac{mx}{R}\cos\frac{my}{R} + \frac{3}{20}f_1 f_2 m^2\cos\frac{2mx}{R}\cos\frac{my}{R}$$

$$+\frac{3}{20}f_1 f_2 m^2\cos\frac{mx}{R}\cos\frac{2my}{R} + \frac{1}{40}f_1^2 m^2\cos\frac{3mx}{R}\cos\frac{my}{R} + \frac{1}{40}f_1^2 m^2\cos\frac{mx}{R}\cos\frac{3my}{R} + \frac{1}{16}f_1^2 m^2\cos\frac{2mx}{R}\cos\frac{2my}{R}\Big]$$

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$$-\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 = -\frac{1}{2}m^2\frac{1}{4}\Big[f_1^2\Big\{ \frac{1}{4}\Big(1+\cos\frac{2my}{R}\Big)\Big(1-\cos\frac{2my}{R}\Big) + \frac{1}{2}\cos\frac{my}{R}\Big(\cos\frac{my}{R} - \cos\frac{3my}{R}\Big)$$

$$+\frac{1}{8}\Big(1-\cos\frac{4my}{R}\Big)\Big\} + \frac{1}{2}f_2^2\Big(1-\cos\frac{2my}{R}\Big) + \cdots\Big]$$

$$= -\frac{1}{8}m^2\Big[\frac{3}{8}f_1^2 + \frac{1}{2}f_2^2\Big] + \cdots$$

$$\frac{\lambda}{E} + \nu\frac{\sigma}{E} - \frac{3}{64}f_1^2 m^2 - \frac{1}{16}f_2^2 m^2 + \Big(f_0 + \frac{1}{4}f_1\Big) = 0$$

$$\boxed{\frac{\lambda}{E} = \frac{3}{64}f_1^2 m^2 + \frac{1}{16}f_2^2 m^2 - \Big(f_0 + \frac{1}{4}f_1\Big) - \nu\frac{\sigma}{E}}$$

$$\wp_1 = -8\frac{\sigma}{E}\Big[\frac{\sigma}{E} + \nu\frac{\lambda}{E} + \frac{3}{64}f_1^2 m^2 + \frac{1}{16}f_2^2 m^2\Big]$$

$$= -4\Big[2(1-\nu^2)\Big(\frac{\sigma}{E}\Big)^2 + (1+\nu)\frac{3}{32}\frac{\sigma}{E}f_1^2 m^2 + (1+\nu)\frac{1}{8}\frac{\sigma}{E}f_2^2 m^2 - 2\nu\Big(f_0 + \frac{1}{4}f_1\Big)\Big]$$

$$4\Big[\Big(\frac{\sigma}{E}\Big)^2 + \Big(\frac{\lambda}{E}\Big)^2 + 2\nu\frac{\sigma}{E}\frac{\lambda}{E}\Big] = 4\Big[(1+\nu^2)\Big(\frac{\sigma}{E}\Big)^2 + \frac{9}{4096}f_1^4 m^4 + \frac{1}{256}f_2^4 m^4 + \Big(f_0 + \frac{1}{4}f_1\Big)^2 + \frac{3}{512}f_1^2 f_2^2 m^4$$

$$-\frac{3}{32}f_1^2 m^2\Big(f_0 + \frac{1}{4}f_1\Big) - \cancel{\frac{3}{32}\nu\frac{\sigma}{E}f_1 m^2} - \frac{1}{8}f_2^2 m^2\Big(f_0 + \frac{1}{4}f_1\Big) - \cancel{\frac{1}{8}\nu\frac{\sigma}{E}f_2 m^2} + \cancel{2\nu\frac{\sigma}{E}\Big(f_0 + \frac{1}{4}f_1\Big)}$$

$$+\cancel{\frac{3}{32}\nu\frac{\sigma}{E}f_1^2 m^2} + \cancel{\frac{1}{8}\nu\frac{\sigma}{E}f_2^2 m^2} - \cancel{2\nu\frac{\sigma}{E}\Big(f_0 + \frac{1}{4}f_1\Big)} - 2\nu^2\Big(\frac{\sigma}{E}\Big)^2\Big]$$

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$$= 4\left[(1-\nu^2)\left(\frac{\sigma}{E}\right)^2 + \frac{9}{4096} f_1^4 m^4 + \frac{1}{256} f_2^4 m^4 + \left(f_0 + \frac{1}{4} f_1\right)^2 + \frac{3}{512} f_1^2 f_2^2 m^4 - \left(\frac{3}{32} f_1^2 m^2 + \frac{1}{8} f_2^2 m^2\right)\left(f_0 + \frac{1}{4} f_1\right)\right]$$

$$K = -4\left[(1-\nu^2)\left(\frac{\sigma}{E}\right)^2 + (1+\nu)\frac{3}{32}\frac{\sigma}{E} f_1^2 m^2 + (1+\nu)\frac{1}{8}\frac{\sigma}{E} f_2^2 m^2 - 2\nu\left(f_0 + \frac{1}{4} f_1\right)\frac{\sigma}{E}\right.$$

$$\left. - \frac{9}{4096} f_1^4 m^4 - \frac{1}{256} f_2^4 m^4 - \left(f_0 + \frac{1}{4} f_1\right)^2 - \frac{3}{512} f_1^2 f_2^2 m^4 + \left(\frac{3}{32} f_1^2 m^2 + \frac{1}{8} f_2^2 m^2\right)\left(f_0 + \frac{1}{4} f_1\right)\right]$$

$$\frac{\partial K}{\partial f_0} = 0 \qquad \boxed{-2\nu\frac{\sigma}{E}\ 2\left(f_0 + \frac{1}{4} f_1\right) + \left(\frac{3}{32} f_1^2 m^2 + \frac{1}{8} f_2^2 m^2\right) = 0}$$

$$K = -4\left[(1-\nu^2)\left(\frac{\sigma}{E}\right)^2 + (1+\nu)\frac{3}{32}\frac{\sigma}{E} f_1^2 m^2 + (1+\nu)\frac{1}{8}\frac{\sigma}{E} f_2^2 m^2 - \frac{9}{4096} f_1^4 m^4 - \frac{1}{256} f_2^4 m^4 - \frac{3}{512} f_1^2 f_2^2 m^4\right.$$

$$\left. + \left(f_0 + \frac{1}{4} f_1\right)^2\right]$$

$$= -4\left[(1- \left(\frac{\sigma}{E}\right)^2 + \frac{3}{32}\frac{\sigma}{E} f_1^2 m^2 + \frac{1}{8}\frac{\sigma}{E} f_2^2 m^2\right]$$

$$\boxed{K = - \quad 4\left(\frac{\sigma}{E}\right)^2 - \frac{3}{8}\frac{\sigma}{E} f_1^2 m^2 - \frac{1}{2}\frac{\sigma}{E} f_2^2 m^2}$$

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$$\mathcal{P}_2 = \frac{1}{4}\Big[2\left(\frac{1}{4} f_1 m^2 - 1\right)^2 f_2^2 + \frac{1}{8} f_1^2 f_2^2 m^4 + \frac{1}{8} f_1^2 \left(\frac{1}{4} f_1 m^2 - 1\right)^2 + \frac{1}{128} f_1^4 m^4 + \frac{1}{4}\left(\frac{1}{2} f_1 m^2 + \frac{1}{2} f_2^2 m^2 - f_1\right)^2$$

$$+ \frac{18}{400} f_1^2 f_2^2 m^4 + \frac{1}{800} f_1^4 m^4 + \frac{1}{256} f_1^4 m^4 \Big]$$

$$= \frac{1}{4}\Big[\frac{1}{8} f_1^2 f_2^2 m^4 - f_1 f_2^2 m^2 + 2 f_2^2 + \frac{1}{8} f_1^2 f_2^2 m^4 + \frac{1}{128} f_1^4 m^4 - \frac{1}{16} f_1^3 m^2 + \frac{1}{8} f_1^2 + \frac{1}{128} f_1^4 m^4$$

$$+ \frac{1}{16} f_1^4 m^4 + \frac{1}{16} f_2^4 m^4 + \frac{1}{4} f_1^2 + \frac{1}{8} f_1^2 f_2^2 m^4 - \frac{1}{4} f_1^3 m^2 - \frac{1}{4} f_1 f_2^2 m^2 + \frac{9}{200} f_1^2 f_2^2 m^4$$

$$+ \frac{1}{800} f_1^4 m^4 + \frac{1}{256} f_1^4 m^4 \Big]$$

$$= \frac{1}{4}\Big[f_1^4 m^4 \left[\frac{1}{800} + \frac{1}{256} + \frac{1}{128} + \frac{1}{128} + \frac{1}{16}\right] + f_1^2 f_2^2 m^4 \left[\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{9}{200}\right] + f_2^4 m^4 \left[\frac{1}{16}\right]$$

$$- f_1^3 m^2 \left(\frac{1}{16} + \frac{1}{4}\right) - f_1^2 f_2 m^2 (0) - f_1 f_2^2 m^2 \left(1 + \frac{1}{4}\right) + \frac{7}{8} f_1^2 + 2 f_2^2 \Big]$$

$$\mathcal{P}_2 = \frac{1}{4}\Big[\frac{533}{6400} f_1^4 m^4 + \frac{21}{50} f_1^2 f_2^2 m^4 + \frac{1}{16} f_2^4 m^4 - \frac{5}{16} f_1^3 m^2 - \frac{5}{4} f_1 f_2^2 m^2 + \frac{3}{8} f_1^2 + 2 f_2^2 \Big]$$

$$\mathcal{P}_3 = \frac{1}{12(1-\nu^2)} \left(\frac{t}{R}\right)^2 \frac{m^4}{4} \Big[(4+2+2) f_1^2 + (2+2) f_2^2 \Big] = \frac{1}{12(1-\nu^2)} \left(\frac{t}{R}\right)^2 m^4 \Big[2 f_1^2 + f_2^2 \Big]$$

$$\frac{3}{4}\frac{\sigma}{E}m^2 = \frac{1}{4}\left[\frac{533}{1600}f_1^2 m^4 + \frac{21}{25}f_2^2 m^4 - \frac{15}{16}f_1 m^2 - \frac{5}{4}\left(\frac{f_2}{f_1}\right)f_2 m^2 + \frac{3}{4}\right] + \frac{1}{3(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4$$

$$\frac{\sigma}{E}m^2 = \frac{1}{4}\left[\frac{21}{25}f_1^2 m^4 + \frac{1}{4}f_2^2 m^4 - \frac{5}{2}f_1\ m^2 + 4\right] + \frac{1}{6(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4$$

$$\frac{\sigma}{E}m^2 = \frac{533}{4800}f_1^2 m^4 + \frac{7}{25}f_1^2 m^4 \rho^2 - \frac{5}{16}f_1 m^2 - \frac{5}{12}f_1 m^2 \rho^2 + \frac{1}{4} + \frac{4}{9(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4$$

$$\frac{\sigma}{E}m^2 = \frac{21}{100}f_1^2 m^4 + \frac{1}{16}f_1^2 m^4 \rho^2 - \frac{5}{8}f_1 m^2 + 1 + \frac{1}{6(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4$$

$$\frac{\sigma}{E}m^2 = \left(\frac{7}{25}f_1^2 m^4 - \frac{5}{12}f_1 m^2\right)\rho^2 + \frac{533}{4800}f_1^2 m^4 - \frac{5}{16}f_1 m^2 + \frac{1}{4} + \frac{4}{9(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4$$

$$\frac{\sigma}{E}m^2 = \left(\frac{1}{16}f_1^2 m^4\right)\rho^2 \qquad + \frac{21}{100}f_1^2 m^4 - \frac{5}{8}f_1 m^2 + 1 + \frac{1}{6(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4$$

$$\left(\frac{87}{400}f_1^2 m^4 - \frac{5}{12}f_1 m^2\right)\frac{\sigma}{E}m^2 = \left(\frac{7}{25}f_1 m^4 - \frac{5}{12}f_1 m^2\right)\left(\frac{21}{100}f_1^2 m^4 - \frac{5}{8}f_1 m^2 + 1\right)$$

$$- \frac{1}{16}f_1^2 m^4\left(\frac{533}{4800}f_1^2 m^4 - \frac{5}{16}f_1 m^2 + \frac{1}{4}\right) + \frac{1}{3(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4\left(\frac{17}{300}f_1^2 m^4 - \frac{5}{24}f_1 m^2\right)$$

$$\rho = \frac{f_2}{f_1}$$

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$$\left(\frac{87}{100}f_1 m^2 - \frac{5}{3}\right)\frac{\sigma}{E}m^2 = \left(\frac{7}{25}f_1 m^2 - 1\right)\left(\frac{21}{25}f_1^2 m^4 - \frac{5}{2}f_1 m^2 + 4\right) - \frac{1}{4}f_1 m^2\left(\frac{533}{4800}f_1^2 m^4 - \frac{5}{16}f_1 m^2 + \frac{1}{4}\right)$$

$$+ \frac{1}{3(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4\left(\frac{17}{75}f_1 m^2 - \frac{5}{6}\right)$$

$$\frac{147}{625}f_1^3 m^6 - \frac{35}{50}f_1^2 m^4 + \frac{28}{25}f_1 m^2$$

$$- \frac{42}{50}f_1^2 m^4 + \frac{5}{2}f_1 m^2 - 4$$

$$- \frac{533}{19200}f_1^3 m^6 + \frac{5}{64}f_1^2 m^4 - \frac{1}{16}f_1 m^2$$

$$\left(\frac{87}{100}f_1 m^2 - \frac{5}{3}\right)\frac{\sigma}{E}m^2 = \left\{\frac{99571}{480000}f_1^3 m^6 - \frac{2339}{1600}f_1^2 m^4 + \frac{1423}{400}f_1 m^2 - 4\right\} + \frac{1}{3(1-\nu^2)}\left(\frac{t}{R}\right)^2 m^4\left(\frac{17}{75}f_1 m^2 - \frac{5}{6}\right)$$

$$\left(\frac{87}{100}\left(\frac{\sigma}{E}\right)\gamma - \frac{5}{3}\right)\left(\frac{\sigma R}{Et}\right)\gamma = \left\{\frac{99571}{480000}\left(\frac{\sigma}{E}\right)^3\gamma^3 - \frac{2339}{1600}\left(\frac{\sigma}{E}\right)^2\gamma^2 + \frac{1423}{400}\left(\frac{\sigma}{E}\right)\gamma - 4\right\} + \frac{1}{3(1-\nu^2)}\gamma^2\left(\frac{17}{75}\left(\frac{\sigma}{E}\right)\gamma - \frac{5}{6}\right)$$

where $\gamma = \left(m^2\frac{t}{R}\right)$, Put $\left(\frac{\sigma}{E}\right) = \eta$

$$m = \sqrt{\gamma\frac{R}{E}}$$

634

$$\left(\frac{\sigma R}{Et}\right) = \frac{\left\{\frac{99571}{480000}\eta^3\gamma^3 - \frac{2339}{1600}\eta^2\gamma^2 + \frac{1423}{400}\eta\gamma - 4\right\} + \frac{1}{3(1-\nu^2)}\gamma^2\left(\frac{17}{75}\eta\gamma - \frac{5}{6}\right)}{\gamma\left(\frac{87}{100}\eta\gamma - \frac{5}{3}\right)}$$

$$\left(\frac{87}{100}\eta\gamma - \frac{5}{3}\right)\left\{\frac{99571}{160000}\eta^3\gamma^3 - \frac{2339}{800}\eta^2\gamma^2 + \frac{1423}{400}\eta\gamma + \frac{1}{3(1-\nu^2)}\gamma^2\left(\frac{17}{25}\eta\gamma - \frac{5}{3}\right)\right\}$$

$$-\left(\frac{87}{50}\eta\gamma - \frac{5}{3}\right)\left\{\frac{99571}{480000}\eta^3\gamma^3 - \frac{2339}{1600}\eta^2\gamma^2 + \frac{1423}{400}\eta\gamma - 4 + \frac{1}{3(1-\nu^2)}\gamma^2\left(\frac{17}{75}\eta\gamma - \frac{5}{6}\right)\right\} = 0$$

$$\frac{87}{100}\eta\gamma\left\{\frac{99571}{480000}\eta^3\gamma^3 - 0 - \frac{1423}{400}\eta\gamma + 8 + \frac{1}{3(1-\nu^2)}\gamma^2\left(\frac{17}{75}\eta\gamma\right)\right\}$$

$$-\frac{5}{3}\left\{\frac{99571}{240000}\eta^3\gamma^3 - \frac{2339}{1600}\eta^2\gamma^2 + 4 + \frac{1}{3(1-\nu^2)}\gamma^2\left(\frac{34}{75}\eta\gamma - \frac{5}{6}\right)\right\} = 0$$

$$\frac{8.662677}{48.000000}\eta^4\gamma^4 - \frac{123801}{40000}\eta^2\gamma^2 + \frac{696}{100}\eta\gamma + \frac{1}{3(1-\nu^2)}\gamma^2\left(\frac{1479}{7500}\eta^2\gamma^2\right)\}$$

$$-\left\{\frac{99571}{144000}\eta^3\gamma^3 - \frac{2339}{960}\eta^2\gamma^2 + \frac{20}{3} + \frac{1}{3(1-\nu^2)}\gamma^2\left(\frac{34}{45}\eta\gamma - \frac{25}{18}\right)\right\} = 0$$

$$0 = 0.180472\,\eta^4\gamma^4 - 0.691465\,\eta^3\gamma^3 - 0.658567\,\eta^2\gamma^2 + 6.96\,\eta\gamma - 6.666666 + \frac{1}{2.73}\gamma^2\left(0.1972\,\eta^2\gamma^2 - 0.755555\,\eta\gamma + 1.388889\right)\}$$

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$$(0.180472\,\eta^4 + 0.0722344\,\eta^2)\gamma^4 - (0.691465\,\eta^3 + 0.276760\,\eta)\gamma^3$$

$$-(0.658567\,\eta^2 - 0.508751)\gamma^2 + (6.96\,\eta)\gamma - 6.66667 = 0$$

$$1811.94\,\gamma^4 - 694.233\,\gamma^3 - 66.3655\,\gamma^2 + 69.6\,\gamma - 6.666667 = 0$$

$$F(\gamma) = \gamma^4 - 0.383144\,\gamma^3 - 0.0316268\,\gamma^2 + 0.0384119\,\gamma - 0.00367930 = 0$$

$$F'(\gamma) = 4\gamma^3 - 1.149432\,\gamma^2 - 0.0732536\,\gamma + 0.0384119$$

$$F(0.\underset{1375}{124}) = +0.00002651 \qquad F'(0.124) = 0.01928$$

$$F(0.122625) = -0.00000016$$

$$F(0.122633) = 0 \qquad \gamma = 0.122633$$

$$G(\gamma) = \gamma^3 - 0.260511\,\gamma^2 - 0.0685740\,\gamma + 0.0300025 = 0$$

$$G'(\gamma) = 3\gamma^2 - 0.521022\,\gamma - 0.0685740$$

$$G'(\gamma) = 0 = \gamma^2 - 0.173674\,\gamma - 0.0228580$$

$$\gamma = 0.086837 \pm \sqrt{(0.086837)^2 + 0.0228580}$$

$$= 0.086837 \pm \sqrt{0.0303987}$$

$$= 0.086837 \pm 0.174352 = \begin{matrix} 0.261189 \\ -0.087515 \end{matrix}$$

$$\left(\frac{\sigma R}{Et}\right) = \frac{\left\{0.207440(\eta\gamma)^3 - 1.461875(\eta\gamma)^2 + 3.557500(\eta\gamma) - 4\right\} + \gamma^2\left\{0.0830281(\eta\gamma) - 0.305250\right\}}{\gamma\left(0.870000(\eta\gamma) - 1.6666667\right)}$$

$\underline{\underline{637}}$

$$= \frac{\left\{0.207440 \times 1.84426 - 1.461875 \times 1.50389 + 3.557500 \times 1.22633 - 4\right\} - 0.0150389 \times 0.20343}{-0.122633 \times 0.599760}$$

Calculation is incorrect !!!

$$= \frac{1.45632}{0.073550}$$

20 !!!

$\frac{1}{6} - \frac{4}{9}$

$= \frac{1}{3}\left(\frac{1}{2} - \frac{4}{3}\right)$

$3 - 8$

$$\left\{\frac{87}{400}(\eta\gamma)^2 - \frac{5}{12}\eta\gamma\right\}\rho^2 = \frac{475}{4800}(\eta\gamma)^2 - \frac{5}{16}(\eta\gamma) + \frac{3}{4} - \frac{5}{18(1-\nu^2)}\gamma^2$$

$$\rho^2 = \frac{0.09895833(\eta\gamma)^2 - 0.3125(\eta\gamma) + 0.75 - 0.101750\gamma^2}{(\eta\gamma)\left\{0.87(\eta\gamma) - 0.4166667\right\}}$$

$$= \frac{0.514064}{0.797409} = 0.644668$$

$$\rho = 0.802912$$

［Ｇｅｎｅｒａｌ　Ｉｎｆｏｒｍａｔｉｏｎ］
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